Consumer-Optimal Disclosure with Costly Information Acquisition

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Abstract

I study the question of how much product information should be available to consumers. A monopolist sells one unit of product. The consumer is initially uninformed of the product value but can incur costs to observe a noisy signal of his valuation. I show that if it is costly to acquire information, consumer surplus can be increasing in the informativeness of the signal, because the seller sets a lower price to deter the consumer's learning. I also show that there is a positive level of information acquisition cost that maximizes both consumer and total surplus.

1 Introduction

Consumers often acquire product information prior to trade. For example, they may visit online shopping websites and read product descriptions or go to brick and mortar stores to see product samples. These activities are costly but enable consumers to form estimates of their willingness to pay. Now, suppose that, say, a new product review website appears. It is still costly for consumers to visit the website and process relevant information; however, the website enables consumers to form more accurate estimates of their valuations. How does this new information environment affect the welfare of sellers and consumers?

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To answer the question, I consider a monopoly pricing model with costly information acquisition: The seller holds one unit of product, which she does not value. The seller posts a price as a take-it-or-leave-it offer. After observing the price, the consumer decides whether to incur costs to observe a noisy signal of his valuation, and then he makes a purchase decision. The main focus is on how equilibrium depends on the informativeness of the signal and the cost to acquire it.

The first finding is that if it is costly for the consumer to acquire information, a more informative signal can decrease the product price and increase consumers surplus, *even though the consumer does not observe the signal in equilibrium.* The logic is as follows. If information acquisition is costly, the seller prefers to set a low price to induce the consumer to buy the product without acquiring information. If the signal becomes more informative, the consumer has a greater incentive to observe it. This implies that the seller has to set an even lower price to deter learning. Thus, the greater availability of product information could benefit consumers even if they do not acquire it.

A natural question is, in the first place, whether the cost of information acquisition hurts consumers. The first result does not answer this question because I fix the cost and vary only the informativeness of the signal. Indeed, one might think that higher costs hurt consumers because uninformed consumers may buy products whose values fall under prices. However, I show that both consumer and total surplus are typically maximized when the consumer incurs positive costs to acquire information. Intuitively, when the consumer can acquire information cheaply but not freely, the seller prefers to lower the price to deter learning. I show that the consumer's benefit from low prices exceeds the cost of potentially buying the product with low values.

This work relates to two strands of existing literature: information disclosure and mechanism design with information acquisition. For example, Lewis and Sappington (1994), Johnson and Myatt (2006) and Ivanov (2013) study sellers' incentives to disclose information to buyers. This paper differs from their works in three aspects: First, the consumer in my model has to incur costs to observe information. Second, I focus on an information structure maximizing consumer surplus, which would be relevant when we consider regulators or online platforms. Third, I consider all Blackwell experiments of the consumer's valuation instead of a particular class of signals. This paper also relates to Wang (2017), which shows that a firm has an incentive to disclose partial information to deter consumers from searching for additional information. In Wang (2017), consumers freely acquire information provided by the firm, and they can subsequently incur costs to be perfectly informed about their values. In my model, there is no free information, and the main focus is on the quality of information that consumers can acquire at costs.

Second, the paper relates to works on mechanism design with information acquisition in economics and marketing literature (Crémer and Khalil, 1992; Crémer et al., 1998a,b; Szalay, 2009; Guo and Zhang, 2012; Shi, 2012; Terstiege, 2016; Ye and Zhang, 2017; Lagerlöf and Schottmüller, 2018a,b). The literature considers richer settings such as auctions and nonlinear pricing, and focuses on the characterization of optimal mechanisms. Two exceptions are Lagerlöf and Schottmüller (2018a,b), which conduct local comparative statics in search costs in a monopoly insurance model. They show that, in a numerical example, a positive search cost can maximize consumer surplus. I complement this literature by showing richer comparative statics in a simpler setting. Finally, Roesler and Szentes (2017) studies a model in which the consumer commits to an information acquisition policy to influence the seller's pricing. Later I show how their result can change if learning is costly.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 characterizes equilibrium given a signal structure and a cost to observe signal realizations. Section 4 studies the consumer-optimal signal given cost, and Section 5 studies the consumer-optimal cost given a signal structure. Section 6 considers consumers with heterogeneous information acquisition costs, and Section 7 concludes.

2 Model

The model consists of a seller, a consumer, and an information designer. The seller holds one unit of product, and her valuation is zero. The consumer's valuation is $v \in \mathbb{R}$ drawn from the cumulative distribution function F, which is commonly known to all players. For notational simplicity, I define $F(x)$ as the probability of $v < x$. F satisfies $F(0) = 0$ and has a mean of $\mu := \int_{-\infty}^{+\infty} v dF(v) > 0$. Thus, it is always efficient to trade.

A *signal* (S, σ) is a pair of a signal realization space S and a function $\sigma : v \mapsto \sigma(v) \in \Delta(S)$, where $\Delta(S)$ is the set of all probability distributions over S. Hereafter, I often write a signal as σ instead of (S, σ). I define the *fully informative signal* as a signal σ^* that reveals v, e.g., $\sigma^*(v)$

draws a realization $s = v$ with probability 1 for any possible v.

The timing of the game is as follows. First, the designer publicly chooses a signal σ from the set of all signals. Second, the seller sets price p, which the consumer observes. Then, Nature draws the value $v \sim F$ and a signal realization $s \sim \sigma(v)$. At this point, neither the seller nor the consumer observes (v, s). The consumer then chooses whether to observe s. If the consumer chooses to observe, he incurs a cost of $c \geq 0$ and observes s.[1] Otherwise, he does not incur any costs and does not observe s. Finally, the consumer decides whether to buy the product.

The payoff of each player is as follows. Let $\gamma = 0$ if the consumer does not observe a signal realization, and $\gamma = c$ if he does. If the consumer purchases the product, his ex post payoff is $v - p - \gamma$; otherwise, his ex post payoff is $-\gamma$. The seller's payoff is her revenue. Both the consumer and the seller are risk neutral. I do not specify the preferences of the designer for now, because I study how the equilibrium depends on the designer's strategy.

The solution concept is subgame perfect equilibrium in which the seller breaks ties in favor of the consumer when she sets a price.[2] Whenever it is clear from context, I use "equilibrium" to mean subgame perfect equilibrium of a subgame that starts from a node at which the seller sets a price. Note that if the consumer does not acquire information, he buys the product if and only if $\mu \geq p$. If the consumer observes s, he compares p with the expected valuation conditional on s.

I briefly discuss the interpretation of the designer's strategy and the consumer's information acquisition. We may view the designer as a regulator or an online platform which chooses a regulation or a platform design to affect what product information is available to consumers. We can also think of different strategies of the designer as different information environments. For instance, a more informative signal might correspond to a situation in which the consumer can access a new customer review website.

The consumer's information acquisition can be investing time and cognitive resources in reading product descriptions and reviews; alternatively, it can be a choice between buying products (say clothes) on the Internet or visiting a brick and mortar store to try them on. The latter is more costly but provides more information to consumers.

[1] Hereafter, I use "information acquisition," "learning s," and "inspection" interchangeably.

[2] As is common in this type of game, we do not need perfect Bayesian equilibrium because the designer chooses a signal before observing a realized valuation.

3 Equilibrium Analysis with a Fixed Signal

In this section, I fix a signal σ of the designer and solves the corresponding subgame between the seller and the consumer. I solve the game in three steps. First, I derive the seller's optimal price that deters information acquisition. Second, I derive the highest price such that the consumer prefers to acquire information rather than exiting the market. Using these prices, I characterize an equilibrium.

Any signal σ induces a distribution over posterior beliefs about v. Moreover, because the consumer is risk neutral, it is enough to keep track of the distribution of posterior means. Thus, given a signal σ, let G denote the distribution of posterior expectations induced by σ and F. Note that even if F has nice properties such as differentiability or concavity of $p[1 - F(p)]$, G may fail to have these properties because I do not impose any restrictions on σ.[3]

Given price $p \leq \mu$, the consumer prefers not to learn s if and only if

$$\begin{aligned} & \mu - p \geq \int_0^{+\infty} \max(x - p, 0) dG(x) - c \qquad (1) \\ \iff & 0 \geq \int_0^{+\infty} \max(0, p - x) dG(x) - c \\ \iff & c \geq \int_0^{p} p - x dG(x) \\ \iff & c \geq \int_0^{p} G(x) dx. \end{aligned}$$

I obtain the second inequality by subtracting $\mu - p = \int_0^{+\infty} x dG(x) - p$ from both sides of the first inequality. The last inequality follows from the integration by parts.

For each $c > 0$, define $x_0(c)$ implicitly by $c = \int_0^{x_0(c)} G(x)dx$. For $c = 0$, this equation does not uniquely determine $x_0(c)$, so I define $x_0(0)$ as the infimum of the support of G. I sometimes write $x_0(c)$ as x_0 for simplicity or as $x_0(c, G)$ to emphasize that $x_0(c)$ also depends on G. Note that if c is sufficiently high so that $x_0(c) \geq \mu$ holds, the seller can extract full surplus by charging price μ. Thus, we obtain the following.

Lemma 1. *Among the prices under which the consumer buys the product without observing a signal, price* $p_0(c) := \max(x_0(c), \mu)$ *uniquely maximizes revenue, where* $x_0(c)$ *satisfies* $c =$

[3] For example, G is a step function if signal σ only reveals whether the value exceeds some cutoff.

$\int_0^{x_0(c)} G(x)dx$ *for any* $c \geq 0$.

Second, I derive the highest price that makes it individually rational for the consumer to observe a signal realization. Note that if the consumer is willing to learn s given price $p^M(G) := \min\left(\arg\max_p p[1 - G(p)]\right)$, the seller prefers to set $p^M(G)$. In contrast, if cost c is so high that the consumer obtains a negative payoff by acquiring information, the seller has to set a lower price than $p^M(G)$.

Note that given price p, the consumer prefers to learn s rather than no purchase if and only if

$$\begin{aligned} & \int_0^{+\infty} \max(x - p, 0) dG(x) - c \geq 0 \\ \iff & \int_p^{+\infty} (x - p) dG(x) \geq c \\ \iff & \int_p^{+\infty} 1 - G(x) dx \geq c. \end{aligned} \tag{2}$$

For $c > 0$, define $x_1(c)$ implicitly by $c = \int_{x_1(c)}^{+\infty} 1 - G(x)dx$. Also, let $x_1(0)$ be the supremum of the support of G. Given price p, the consumer prefers to observe a signal rather than exiting the market if and only if $p \leq x_1(c)$. Note that depending on c, the consumer may buy the product without inspection at price $x_1(c)$.

Lemma 2. *Define* $p_1(c) := \min\left(\arg\max_{p \leq x_1(c)} p[1 - G(p)]\right)$. *Then,* $p_1(c)$ *is the (lowest) optimal price subject to the constraint that the consumer prefers to observe a signal rather than exiting the market.*[4]

I define a cutoff c^* as follows. $p_0(c)$ in Lemma 1 is continuous, and it is strictly increasing in c until it hits μ. Also, $p_1(c)[1 - G(p_1(c))]$ is continuous and decreasing in c.[5] Moreover, it holds that $p_0(0) = x_0(0) \leq p_1(0)[1 - G(p_1(0))] \leq \mu = \lim_{c \to +\infty} p_0(c)$. Let c^* denote the unique level of the cost satisfying

$$p_0(c^*) = p_1(c^*)[1 - G(p_1(c^*))]. \tag{3}$$

[4] If $1 - G$ is log-concave, $p_1(c)$ simplifies to $\min(p^M(G), x_1(c))$. However, $1 - G$ fails to be log-concave for a wide range of signals, even if the underlying value distribution F has nice properties.

[5] The continuity holds for the following reason: If $p_1(c)[1 - G(p_1(c))]$ is discontinuous at some c', then there is $\delta > 0$ such that for any $\varepsilon > 0$, $\max_{p \leq x_1(c')} p[1 - G(p)] - \max_{p \leq x_1(c'+\varepsilon)} p[1 - G(p)] > \delta$. $p_1(c') = x_1(c')$ holds, because if $p_1(c') < x_1(c')$, $p_1(c') \leq x_1(c' + \varepsilon)$ holds for a sufficiently small $\varepsilon > 0$. However, if the seller sets price $x_1(c' + \varepsilon)$ given cost $c' + \varepsilon$, then her revenue $p(x_1(c' + \varepsilon))[1 - G(x_1(c' + \varepsilon))]$ converges to $p(x_1(c')[1 - G(x_1(c'))]$ as $\varepsilon \to 0$ because $p[1 - G(p)]$ is left-continuous and $x_1(c' + \varepsilon)$ converges to $x_1(c')$ from below as $\varepsilon \to 0$.

If information acquisition cost is c^*, the seller is indifferent between deterring the consumer's learning by charging $p_0(c^*)$ and inducing it by charging $p_1(c^*)$.

Proposition 1. *In equilibrium, the consumer acquires information if and only if $c < c^*$. If $c < c^*$, the seller sets a price of $p_1(c)$. If $c \geq c^*$, the seller sets a price of $p_0(c) \leq p_1(c)$.*

Proof. First, $c \geq c^*$ implies $p_0(c) \geq p_1(c)[1 - G(p_1(c))]$. This implies that the seller prefers to set $p_0(c)$ to deter learning, because if the seller chooses another price and the consumer learns s, the seller's revenue is at most $p_1(c)[1 - G(p_1(c))]$. (The seller's tie-breaking rule implies that she sets $p_0(c^*) \leq p_1(c^*)$ at $c = c^*$.) Second, if $c < c^*$, $p_0(c) < p_1(c)[1 - G(p_1(c))]$. This implies $p_0(c) < p_1(c)$, which implies that the consumer prefers to learn s given price $p_1(c)$. Thus, $p_1(c)$ maximizes revenue among all prices that induce learning. Therefore, if $c < c^*$, the seller posts price $p_1(c)$ and the consumer observes a signal realization. $\square$

Before proceeding to the comparative statics, I summarize the equilibrium payoffs.

Corollary 1. *In equilibrium, the following holds.*

- *If $c < c^*$, the seller and the consumer obtain expected payoffs of $p_1(c)[1 - G(p_1(c))]$ and $\int_{p_1(c)}^{+\infty} 1 - G(x)dx - c$, respectively.*
- *If $c \geq c^*$, the seller and the consumer obtain expected payoffs of $p_0(c)$ and $\mu - p_0(c)$, respectively.*

4 Optimal Signal with Fixed Cost

This section considers the following question: To enhance consumer or total surplus, how much and what kind of information should the designer disclose? This is a difficult problem because we have to consider all signals (i.e., all Blackwell experiments of $v \sim F$) and study how they affect the distributions of posterior means and prices. Indeed, the case of $c = 0$ was recently solved by Roesler and Szentes (2017), which shows that partially informative signals often maximize consumer surplus.

The following result shows that, however, full disclosure is optimal if it is significantly costly to acquire information. To state the result, let c^* be the cutoff in Proposition 1. Namely, c^*, which satisfies $p_0(c^*, F) = p_1(c^*, F)[1 - F(p_1(c^*, F))]$, makes the seller indifferent between deterring and inducing the consumer's learning, when the designer chooses the fully informative signal (whose distribution of posterior means is F).

Theorem 1. *If $c \geq c^*$, among all signals, the fully informative signal maximizes consumer surplus and total surplus, and it minimizes the price and the seller's revenue. In equilibrium, the consumer does not acquire information.*

Proof. Fix any $c \geq c^*$; that is, the consumer does not acquire information under the fully informative signal σ^*. Take an arbitrary signal σ, and let G denote the distribution of posterior means under σ. It holds that F is a mean-preserving spread of G. I show $p_0(F) \leq p_0(G)$. Indeed, $x_0(F)$ and $x_0(G)$ satisfy $c = \int_0^{x_0(F)} F(x)dx = \int_0^{x_0(G)} G(x)dx$. Because F is a mean-preserving spread of G, $\int_0^x F(v)dv \geq \int_0^x G(v)dv$ for any $x \geq 0$. Thus, $x_0(F) \leq x_0(G)$, which implies $p_0(F) = \min(x_0(F), \mu) \leq \min(x_0(G), \mu) = p_0(G)$.

There are two cases to consider. If the consumer does not acquire information under σ, the consumer is better off under σ^* because $\mu - p_0(F) \geq \mu - p_0(G)$. Similarly, the seller's payoff is lower under F. Second, suppose that the consumer acquires information under σ. Then, we obtain $\mu - p_0(F) \geq \mu - p_0(G) > \int_{p_1(G)}^{+\infty} xdG(x) - p_1(G)[1 - G(p_1(G))]$. The last inequality holds because the total surplus is lower but revenue is strictly higher when the seller charges $p_1(G)$ instead of $p_0(G)$. Because the last expression is the consumer's payoff under σ, the consumer obtains a greater payoff under σ^*. In contrast, the seller's payoff is lower under F as $p_0(F) \leq p_0(G) < p_1(G)[1 - G(p_1(G))]$. Finally, σ^* maximizes total surplus because trade occurs with probability 1 and the consumer does not incur information acquisition cost. $\square$

The results have a novel implication on how the availability of product information affects consumer welfare: *The mere availability of product information can benefit consumers even if they do not acquire it.* To see this, suppose that consumers have access to new information sources such as customer review websites, which enable them to acquire more accurate information about products and services. How does the new source of information benefit consumers? A typical argument would be that they can more accurately check whether their valuations exceed prices; alternatively, we might argue that the information could change the distribution of consumers' willingness to pay in a way that sellers lower prices. Theorem 1 points to a new benefit: The new information source can benefit consumers because sellers have an incentive to lower prices to discourage consumers from learning the new information.

Note that Theorem 1 is vacuous for a sufficiently large c. For example, if c is greater than the highest possible value under F, the consumer never acquires information. Then, any signals give him a payoff of zero, and thus the fully informative signal trivially maximizes consumer surplus. Although this paper does not study how small the cost can be for Theorem 1 to apply, the following example suggests that c might not need to be extremely high.

Example 1. Suppose that v is uniformly distributed on $[0, 1]$. If $c = 0$, the consumer's payoff is maximized by partial disclosure (Roesler and Szentes, 2017). In contrast, if $c \geq c^* = 1/32 \approx 0.03$, the consumer's payoff is maximized by full disclosure. (See Footnote 7 for the details.)

Note that full disclosure is typically not a unique signal maximizing consumer welfare, because $p_0(G)$ does not depend on the shape of $G(x)$ for $x \geq \mu$. However, it is not hard to find a natural class of signals that singles out full disclosure as a unique optimum. For example, given any $\eta \in [0, 1]$, consider a *truth-or-noise* signal σ_η that sends $s = v$ with probability η and $s = z$ with probability $1 - \eta$, where z is an independent draw from F. The consumer can observe (η, s) but cannot tell whether s is the true value v or a noise z. Appendix A proves that, if c is greater than but close to c^*, among all truth-or-noise signals, full disclosure ($\eta = 1$) uniquely maximizes consumer surplus. This is because $\int_0^z G_\eta(x)dx$ strictly increases in η for any $p \leq \mu$ and thus $\eta = 1$ uniquely minimizes the equilibrium price $p_0(G_\eta)$. Here, G_η is the distribution of posterior expectations given σ_η.

Next, I establish a more general comparative statics on valuation distributions. Formally, given c, let $\mathcal{N}(c)$ denote the set of distributions under which the seller deters learning:

$$\mathcal{N}(c) := \{G : p_0(c, G) \geq p_1(c, G)[1 - G(p_1(c, G))]\}\,.$$

Precisely, $\mathcal{N}(c)$ is the set of distributions of posterior means such that the consumer does not acquire information in equilibrium. Different distributions in $\mathcal{N}(c)$ might come from different (true) valuation distributions or signals, but this distinction does not matter.

It would be difficult to explicitly characterize distributions in $\mathcal{N}(c)$. However, it is easy to find (c, G) such that $G \in \mathcal{N}(c)$. Indeed, for any distribution G satisfying Assumption 2 in the next section, $p_0(c, G) > p_1(c, G)[1 - F(p_1(c, G))]$ holds for any c close to $c < x_0^{-1}(\mu)$.

Proposition 2. *Fix c and let G and H denote two distributions of the consumer's posterior expectations. If $G \in \mathcal{N}(c)$ and H second-order stochastically dominates G, the seller sets a higher price and obtains a higher payoff under H than G.*

Proof. Note that $p_0(G) \leq p_0(H)$ by the same argument as the previous theorem. First, if the consumer does not observe a signal given H, the seller sets a higher price and obtains a higher revenue under H. Second, if the consumer observes a signal under H, we obtain $p_0(G) \leq p_0(H) \leq p_1(H)[1 - H(p_1(H))] \leq p_1(H)$. That is, the seller sets a higher price and obtain greater revenue under H than G. □

The result is in contrast to the standard monopoly pricing where the first or second order stochastic shift has no implications on monopoly prices. Finally, the effect of the second order stochastic shift on consumer surplus is ambiguous, because the stochastic shift increases both the product price and average willingness to pay.

4.1 Optimal Signal under $c < c^*$

Theorem 1 shows that, if $c \geq c^*$, the greater availability of information could benefit the consumer even if he does not acquire it. I show that this insight can be relevant no matter how small c is. To do so, I impose two simplifying assumptions.

Assumption 1. The valuation distribution F is the uniform distribution on $[0, 1]$, and the designer chooses from the set of all truth-or-noise signals $\{\sigma_\eta\}_{\eta \in [0,1]}$.

The following result states that the designer can maximize consumer surplus by disclosing as much information as possible subject to the constraint that the consumer does *not* acquire the information.

Proposition 3. *Fix any cost $c > 0$ to observe signals. Under Assumption 1, the consumer-optimal outcome involves no information acquisition. Formally, both consumer and total surplus are maximized by σ_{η^*} where $\eta^* \in (0, 1]$ is the highest precision under which the consumer dose not acquire information.*

Proof. The case of $c \geq c^*$ follows from Theorem 1. Thus, assume $c < c^*$. Abusing notation slightly, let $\mathcal{N} \subset [0,1]$ denote the set of all η's such that the consumer does not acquire information given the seller's optimal pricing under σ_η. Note that $c < c^*$ implies $\eta^* := \max \mathcal{N} < 1$. By the same argument as the proof of Theorem 1, among all signals σ_η with $\eta \in [0, \eta^*]$, σ_{η^*} maximizes both consumer and total welfare. To complete the proof, I show that any σ_η with $\eta > \eta^*$ gives the consumer a lower payoff than σ_{η^*}. Fix any $\eta \in (\eta^*, 1]$. By the construction of η^*, given η, the consumer acquires information in equilibrium. To simplify notation, I write (η, c) to mean the problem in which the signal is σ_η and the consumer incurs a cost of c to observe it. Also, let $U(\eta, c)$ denote the consumer's equilibrium payoff at (η, c). I show $U(\eta, 0) \geq U(\eta, c)$. First, suppose that the equilibrium price at (η, c) is $x_1(\eta)$, which is the price that makes the consumer indifferent between acquiring information and exiting the market. By construction, $U(\eta, c) = 0 \leq U(\eta, 0)$. Second, suppose that the equilibrium price at (η, c) is $p^M(\eta) := \max p[1 - G_\eta(p)]$, where G_η is the distribution of posterior means given σ_η. Because the seller continues to set $p^M(\eta)$ even at $(\eta, 0)$, $U(\eta, c) = U(\eta, 0)$. This argument shows $U(\eta, 0) \geq U(\eta, c)$. Thus, it is sufficient to show $U(\eta^*, c) \geq U(\eta, 0)$ for all $\eta > \eta^*$. Appendix B shows that $U(\eta, 0)$ is decreasing in $\eta \geq \eta^*$ by directly calculating $\frac{\partial U}{\partial \eta}$.[6] Thus, we can establish the proposition by showing $U(\eta^*, c) \geq U(\eta^*, 0)$. Total surplus under (η^*, c) is weakly greater than under $(\eta^*, 0)$ because trade occurs for sure and the consumer does not incur c under (η^*, c). Also, the seller weakly prefers $(\eta^*, 0)$ to (η^*, c). Suppose to the contrary that the seller strictly prefers (η^*, c). This implies that the seller strictly prefers to deter the consumer's learning under (η^*, c). As the seller's payoff is continuous in η, we can find a small $\varepsilon > 0$ such that the seller still prefers to deter learning under $(\eta^* + \varepsilon, c)$. This contradicts the definition of η^*. Thus, we can conclude $U(\eta^*, c) \geq U(\eta^*, 0)$, which implies $U(\eta^*, c) \geq U(\eta, c)$ for all $\eta > \eta^*$. Therefore, the consumer's payoff is maximized at (η^*, c) with no information acquisition. $\square$

Figure 1 presents three graphs, which depicts how η affects the consumer's equilibrium payoffs (blue) and the seller's payoffs from $p_0(\eta)$ (orange, dotted) and $p_1(\eta)$ (yellow). Recall that $p_0(\eta)$ is the highest price that deters the consumer's information acquisition, and $p_1(\eta)$ is the highest price such that the consumer prefers information acquisition to exiting the market.

[6] This is the only part that relies on F being the uniform distribution. For any F, the result identical with Proposition 3 holds as long as $U(\eta, 0)$ is decreasing in $\eta \geq \eta^*$.

The graphs not only confirm Proposition 3 but also reveal two observations. First, at each cost c, there is a unique $\eta(c)$ such that the seller induces the consumer's learning if and only if $\eta > \eta(c)$. Second, $\eta(c)$ increases in c. Both observations are intuitive: It is more profitable for the seller to lower prices and deter learning if c is high and η is low, where the consumer has less incentive to observe a signal. Thus, a higher c must be accompanied with a η to keep the seller indifferent between inducing and deterring the consumer's learning.

Finally, the graphs show that the consumer's payoff is maximized at $\eta(c)$ for each c. More precisely, at the optimum, the seller is indifferent between $p_0(\eta(c))$ and $p_1(\eta(c))$, and sets $p_0(\eta(c)) < p_1(\eta(c))$ breaking a tie in favor of the consumer. At price $p_0(\eta(c))$, the consumer strictly prefers not to observe the signal realization.

5 Information Acquisition Costs Benefit Consumer

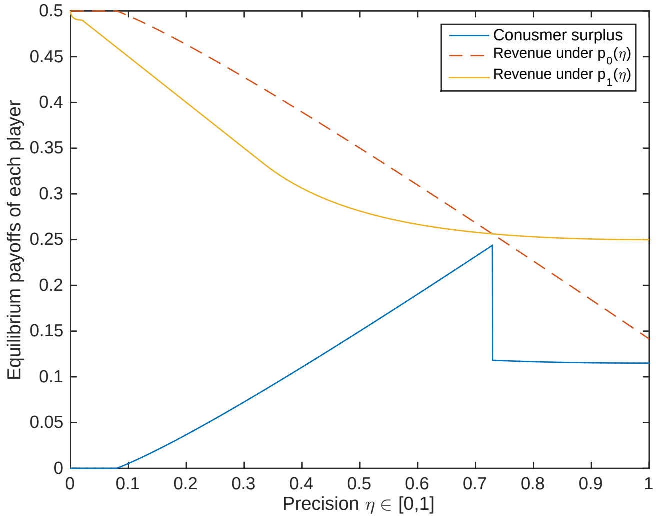
In this section, I fix a distribution G of posterior means and consider comparative statics in the cost c of information acquisition. Figure 2 depicts the consumers' equilibrium payoff as a function of c when G is the uniform distribution on $[0, 1]$.[7] While the consumer's payoff decreases in c below and above $c^* = {}^1\!/_{32}$, it jumps discontinuously at $c^* = {}^1\!/_{32}$, at which the consumer surplus is globally maximized. (In this particular example, the consumer's payoff doubles if the cost increases from 0 to c^*.) I generalize this observation under the following assumption.

Assumption 2. $G(p^M) > 0$, where $p^M := \min(\arg\max_p p[1 - G(p)])$ is the lowest monopoly price under G.

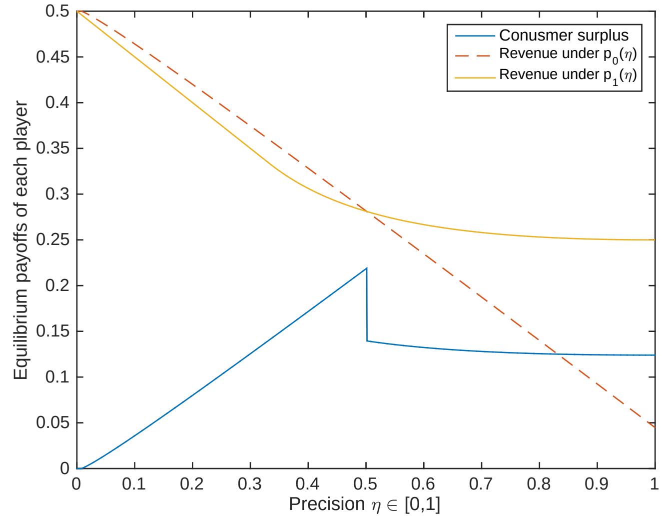
Theorem 2. *Consider any F and σ such that the corresponding distribution of posterior means G satisfies Assumption 2. Then, c^* in equation (3) is positive and uniquely maximizes consumer surplus among all $c \geq 0$. Also, c^* maximizes total surplus and minimizes the seller's revenue. At c^*, the consumer does not acquire information in equilibrium.*

Proof. I show that c^* in Proposition 1 maximizes both consumer and total surplus. First, $c^* > 0$ because Assumption 2 implies $p_0(0) = x_0(0) < p_1(0)[1 - G(p_1(0))]$. Corollary 1 implies that the

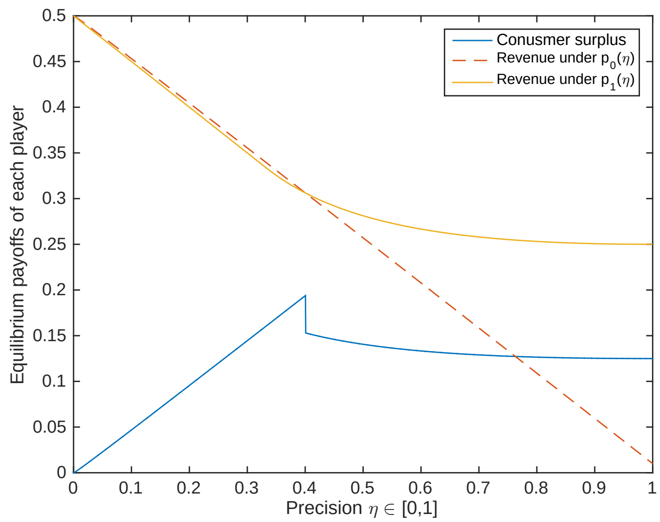
[7] Direct calculations show that the consumer's equilibrium payoff is ${}^1\!/_8 - s$ if $c < c^* = {}^1\!/_{32}$, ${}^1\!/_2 - \sqrt{2s}$ if $s \in [{}^1\!/_{32}, {}^1\!/_8]$, and 0 if $s \geq {}^1\!/_8$.



$c = 0.01$



$c = 0.0001$



$c = 0.00005$

Figure 1: Optimal precision η at $c = 0.01, 0.0001, 0.00005$

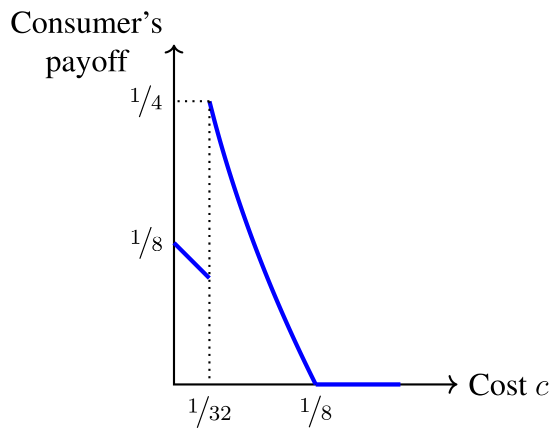


Figure 2: Consumer surplus when $G = U[0, 1]$

consumer's payoff is decreasing in c on $[c^*, +\infty)$. Now, define

$$\hat{c} := \min\left(\arg\max_{c\in[0,c^*]} \int_{p_1(c)}^{+\infty} 1 - G(x)dx - c\right).$$

$\hat{c}$ maximizes consumer surplus among all costs under which the consumer acquires information in equilibrium. If $\hat{c} = c^*$, the proof is done. Suppose $\hat{c} < c^*$. To prove that c^* globally maximizes the consumer's equilibrium payoff, it is enough to show that the consumer's payoff is strictly greater at $c = c^*$ than at $c = \hat{c}$. Suppose that the cost increases from $c = \hat{c}$ to $s = c^*$. The seller's payoff weakly decreases, i.e., $p_1(c^*)[1 - G(p_1(c^*))] \leq p_1(\hat{c})[1 - G(p_1(\hat{c}))]$, because the seller under c^* chooses a price from a smaller set. (See Lemma 2.) However, total surplus strictly increases, because trade is more likely occur and the consumer does not pay the cost at $c = c^*$. Therefore, the consumer is strictly better off at c^* than $\hat{c}$.

Next, c^* maximizes total surplus because the consumer purchases the product with probability 1 and does not incur the information acquisition cost.

Finally, the seller's revenue is minimized at c^*, because it is weakly decreasing in c on $[0, c^*)$, increasing on $[c^*, +\infty)$, and continuous at everywhere. $\square$

Theorem 2 might look similar to the finding of Roesler and Szentes (2017), which shows that a consumer can benefit from being partially ignorant of his valuation.[8] However, they are different:

[8]In a different context, Kessler (1998) finds that the agent could benefit from not being informed of his type in a principal-agent model.

In their work, the seller best responds to the consumer's information acquisition policy. In the current model, the consumer decides whether to learn product value best responding to the price set by the seller. What drives Theorem 2 is the consumer's ability to acquire information relatively cheaply but not freely, and not the fact that he is ignorant of his valuation.

6 Dicussion

6.1 Heterogeneous Costs and Inter-Consumer Externalities

The consumer search literature has investigated inter-consumer externalities between "savvy" consumers, who might be well informed about product values and prices, and "non-savvy" consumers, who might not. (See Armstrong (2015) for the details.) Here, I incorporate heterogeneous information acquisition costs to study inter-consumer externalities.

Suppose that there is a unit mass of consumers. All consumers can access the same signal at costs. Consumer differ in costs they incur to observe signals: The fraction θ of consumers are "savvy" and they do not incur any costs to observe the signal; the fraction $1-\theta$ of consumers are "non-savvy" and incur a cost of $c > 0$ to observe the signal.

To state the first result, fix signal σ, and let G denote the resulting distribution of conditional expectations. The distribution is common between savvy and non-savvy consumers. Assume that $1 - G(\cdot)$ is log-concave.

The following result summarizes the direction of inter-consumer externalities. We say that savvy (non-savvy) consumers benefit from non-savvy (savvy) consumers if the equilibrium payoffs of savvy (non-savvy) consumers are greater than the payoffs under $\theta = 1$ ($\theta = 0$). Recall that $p^M := \min(\arg\max_p p[1-G(p)])$ is the lowest monopoly price given G. Also, p_0 is the highest price among all prices that deter information acquisition by non-savvy consumers.

Proposition 4. *Consider any equilibrium in which non-savvy consumers buy products without acquiring information. Then, the following holds.*

1. *Savvy consumers benefit from non-savvy consumers if and only if* $p^M \geq p_0$.

2. *Non-savvy consumers benefit from savvy consumers if and only if* $p^M \leq p_0$.

Proof. If $p^M \geq p_0$, in any equilibrium in which non-savvy consumers buy products without acquiring information, the seller sets price p_0: Any $p < p_0$ leads to lower revenues from both savvy consumers (due to log-concavity of $1 - G(x)$) and non-savvy consumers; any $p > p_0$ induces non-savvy consumer to acquire information or to give up purchase, which is a contradiction. In the absence of non-savvy consumers, the seller sets price p^M. Combining these observations, we can conclude that savvy consumers benefit from non-savvy consumers whereas non-savvy consumes do not benefit from savvy consumers if $p^M \geq p_0$.

If $p^M \leq p_0$, the seller sets price $p \in [p^M, p_0]$: Any $p < p^M$ leads to lower revenue than p^M, and any $p > p_0$ induces non-savvy consumer to acquire information or to give up purchase, which is a contradiction. Thus, non-savvy consumers benefit from savvy consumers whereas savvy consumes do not benefit from non-savvy consumers if $p^M \leq p_0$. $\square$

In contrast, savvy consumers (weakly) benefit from non-savvy consumers if they acquire information in equilibrium. This is because the seller either sets price p^M or sets a lower price to make it individually rational for non-savvy consumers acquire information. Thus, we obtain the following.

Proposition 5. *Consider any equilibrium in which non-savvy consumers acquire information. Then, savvy consumers benefit from non-savvy consumers.*

Finally, in any equilibrium where non-savvy consumers give up purchase without acquiring information, there is no externality from non-savvy to savvy consumers. In contrast, non-savvy consumers might benefit if savvy consumers do not exist, because the seller might lower price from p^M.

The above results provide a sufficient condition under which savvy consumers benefit from non-savvy consumers:

Corollary 2. *Suppose that G satisfies $p^M \geq \mu$. Then, savvy consumers benefit from non-savvy consumers.*

Which type of equilibrium arises depends on non-savvy consumers' cost c to acquire information and the fraction θ of savvy consumers. For example, decreasing θ moves an equilibrium toward the one in Proposition 4 and Proposition 5 if $c > c^*$ and $c < c^*$, respectively.

6.2 Comparison with Model without Immediate Purchase

The current model differs from models of consumer entry and consumer search in that I allow the consumer to buy the product without learning his willingness to pay. This is a crucial assumption: If consumers have to incur costs not only to learn values but also to purchase products, there is no clear relationship between the informativeness of the signal and equilibrium prices. As simple examples, if G puts probability 1 on $1/2$ and F puts equal probabilities on 0 and 1, the seller sets a higher price under F; if G puts probability 1 on 100 and F puts equal probabilities on 99 and 101, the seller sets a higher price under G. Thus, the more dispersed distribution (in the sense of mean-preserving spread) may increase or decrease equilibrium prices.

7 Conclusion

The main takeaway of this paper is that the greater availability of information can benefit consumers through low prices when it is costly for them to acquire information. I show that this occurs when consumers do not acquire information in equilibrium. Thus, it is not information itself but the "threat" to acquire information that induces the seller to lower prices. The two sets of results formalize this intuition. First, when it is costly for the consumer to observe a noisy signal of his valuations, the fully informative signal could maximize consumer surplus because it gives the consumer the greatest incentive to acquire information, which in turn induces the seller to set a low price. Second, consumer surplus is often maximized at a positive level of information acquisition cost. Thus, the threat to acquire information could benefit the consumer more than the actual information acquisition does.

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Appendix

A Unique Optimality of Full Disclosure Among Truth-or-Noise Signals

I show that full disclosure uniquely maximizes consumer surplus among the truth-or-noise signals, whenever c is in a certain range. Formally, for each $\eta \in [0,1]$, define σ_η as the following signal: A signal realization $s \sim \sigma_\eta(v)$ is $s = v$ with probability η and $s = z$ with probability $1-\eta$, where z is drawn by F independently of the true valuation v.

Proposition 6. *Suppose that F is strictly increasing on its support $[a,b]$ with $a < b$. There is $\delta > 0$ such that for any $c \in [c^*, c^* + \delta]$, full disclosure uniquely maximizes consumer surplus among all the truth-or-noise signals.*

Proof. Given σ_η, the conditional expectation of v given s, denoted by $v(s)$, is $\eta s + (1-\eta)\mu$. Thus, the CDF of $v(s)$ is $G_\eta(x) = \mathbf{P}\left(v(s) \leq t\right) = F\left(\frac{t-(1-\eta)\mu}{\eta}\right)$. Direct calculations show that $G_\eta(x)$ is strictly increasing in η for any $x < \mu$. Thus, $\int_0^p G_\eta(x)dx$ is also strictly increasing in η for any $p \leq \mu$.

Recall that the optimal price deterring information acquisition is $p_0(\eta) := \min(\mu, x_0(\eta))$ where $x_0(\eta)$ satisfies $c = \int_0^{x_0(\eta)} G_\eta(x)dx$. Observe that $x_0(\eta)$ is strictly increasing in η until it hits μ.

Now, take c^* in Theorem 1. If c is close to c^*, we obtain $p_0(1) = x_0(1) < \mu$. Then, for any $\eta < 1$, we obtain $p_0(\eta) > p_0(1)$. By the same argument as the proof of the theorem, we can conclude that the equilibrium price is strictly greater than $p_0(1)$. $\square$

B On the Proof of Proposition 3

I show that $U(\eta,0)$ is decreasing in $\eta \geq \eta^*$. The direct calculation shows that $G_\eta(x) = \frac{x-\frac{1}{2}(1-\eta)}{\eta}$ with support $\left[\frac{1-\eta}{2}, \frac{1+\eta}{2}\right]$. The optimal price $p^M(\eta) = \max_{p\in\left[\frac{1-\eta}{2},\frac{1+\eta}{2}\right]} p\left(1-\frac{x-\frac{1}{2}(1-\eta)}{\eta}\right)$ is

$$p^M(\eta) = \begin{cases} \frac{1-\eta}{2} & \text{if} \quad \eta \leq \frac{1}{3} \\ \frac{1+\eta}{4} & \text{if} \quad \eta \geq \frac{1}{3}. \end{cases} \tag{4}$$

First, I show $\eta^* \geq \frac{1}{3}$. Suppose to the contrary that $\eta^* < \frac{1}{3}$, and take any $\eta \in (\eta^*, \frac{1}{3})$. By construction of η^*, at η, the seller strictly prefers to induce the consumer's learning. However, because $\eta < \frac{1}{3}$, the seller's payoff given learning is at most $\underline{p}[1-G_\eta(\underline{p})]$ where $\underline{p} := \frac{1-\eta}{2}$. Because $\underline{p}$ is the minimum of the support of G_η, the seller is indifferent between learning and no learning, which is a contradiction. This

Given $\eta^* \geq \frac{1}{3}$, for any $\eta \geq \eta^*$, the consumer's payoff is as follows.

$$\begin{aligned}
&U(\eta,0)\\
=&\int_{p^M(\eta)}^{+\infty} 1-G_\eta(x)dx\\
=&\int_{\frac{1+\eta}{4}}^{\frac{1+\eta}{2}} 1-\frac{x-\frac{1}{2}(1-\eta)}{\eta}dx\\
=&\int_{\frac{1+\eta}{4}}^{\frac{1+\eta}{2}} \frac{1}{2}+\frac{1}{2\eta}-\frac{x}{\eta}dx\\
=&\left(\frac{1}{2}+\frac{1}{2\eta}\right)\cdot\left(\frac{1+\eta}{2}-\frac{1+\eta}{4}\right)-\frac{1}{2\eta}\cdot\left\{\left(\frac{1+\eta}{2}\right)^2-\left(\frac{1+\eta}{4}\right)^2\right\}\\
=&\frac{1}{2}\left(\frac{1+\eta}{\eta}\right)\frac{1+\eta}{4}-\frac{1}{2}\cdot\frac{3}{16\eta}(1+\eta)^2\\
=&\frac{(1+\eta)^2}{32\eta}.
\end{aligned}$$

The first derive is $\frac{32(\eta^2-1)}{(32\eta)^2} \leq 0$. Thus, $U(\eta,c)$ is decreasing in $\eta \geq \eta^*$.