Double Marginalization and Misplacement in Online Advertising*

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Abstract

Internet users often visit multiple ad-financed websites as a bundle to fulfill their needs. We ask whether complementary websites have the right incentives to choose their advertising policies. We identify two forces that distort equilibrium away from the industry optimum and the efficient outcome. First, websites place more ads than the industry optimum (double marginalization). Second, given the total advertising volume at equilibrium, websites misallocate ads across websites (misplacement). Perfect competition in one market segment eliminates double marginalization but may exacerbate misplacement. The potential trade-off challenges conventional wisdom that competition would restore the industry optimum. Introducing micropayments removes misplacement, but the welfare consequences are ambiguous. (JEL codes: D21, D40, L23, L42, L86)

Keywords: Platforms, Advertising, Misplacement, Double Marginalization, Competition

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1 Introduction

Internet users often surf multiple websites as complements to fulfill their needs. Some websites play the role of organizers (e.g., search engines, social media platforms, news aggregators), which link their users to content producers (e.g., news media, blogs, product review sites). Many websites do not directly charge users; instead, they "tax" users' attention via advertising. The ads impose nuisance costs on the users and thus may reduce the demand for the service and possibly decrease ad revenue. In such an environment, do complementary websites have the right incentive to choose their advertising policies, either from the perspective of industry profits or social welfare? If websites fail to coordinate their strategies, what changes of market structure or business model might mitigate this?

We study these questions in a game-theoretic model. The model consists of two websites and a mass of consumers. First, each website simultaneously chooses the number of ads to place. Consumers learn their values of visiting websites and decide which websites to visit. For each website they visit, consumers incur a disutility associated with advertising. To capture the complementarity of websites in a stark way, the baseline model assumes that the websites are perfect complements, so that consumers obtain a positive value only by visiting both websites.

We identify two forces that render equilibrium suboptimal in terms of industry profit and total welfare. First, any equilibrium entails the standard *double marginalization* problem: Each website fails to internalize the negative effect that placing ads has on the other website, i.e., the decrease ads cause in users' visit to the that site. As a result, websites place more ads than the industry-optimum or the efficient level. Second, equilibrium typically entails *misplacement* of ads, in that websites could reallocate ads from one site to the other and increase the industry profit, while keeping the consumer's disutility at the equilibrium level. For example, suppose website 1 is more effective than website 2 at converting ads into revenue–e.g., that website 1's ads impose lower disutility on consumers, or its better targeting ability ensures a higher click-through rate.[1] In such a case, any equilibrium entails misplacement, in that the joint profit

[1] Heterogeneous nuisance costs can be equivalently modeled as heterogeneous ads revenue per visit of a user (See Section 6). The literature on online advertising has supported the heterogeneous technology among advertisers in converting consumer attention to ads revenues (e.g., Evans (2008, 2009); Goldfarb and Tucker (2011); Athey et al. (2018)).

would increase if website 1 were to place more ads and website 2 were to place fewer, without changing the total advertising volume.

While double marginalization is well known, misplacement is unique to our model in which websites are heterogeneous in their abilities to convert ads into revenue. The paper clarifies the source of these suboptimal outcomes in terms of the market structure and the instruments that websites can use to monetize consumer visits.

First, the standard argument on the pricing of complements suggests that competition mitigates double marginalization. For example, two monopolists that sell complementary goods—such as tea and sugar— will face double marginalization. If a new firm enters and intensifies competition in the tea market, it will mitigate double marginalization and increase the industry profit (Rey and Tirole, 1986; Shleifer and Vishny, 1993; Lerner and Tirole, 2004, 2015; Dellarocas, 2012). Our model confirms this intuition: Under a certain assumption on consumers' disutility function, competition for one segment (e.g., website 2) eliminates double marginalization and benefits consumers. At the same time, competition may exacerbate misplacement and decrease the industry profit. For example, suppose the equilibrium without competition entails misplacement, in that website 1 places too few ads relative to website 2. If website 1 faces competition, it further decreases the advertising volume, which in turn incentivizes website 2 to place more ads.

Second, we show that misplacement stems from the lack of monetary instruments. Specifically, we augment the model by allowing each website to place ads and charge consumers via per-visit monetary transfers, referred to as "micropayments." The equilibrium in such a game still entails double marginalization, but it no longer has misplacement. We then provide a condition under which the introduction of micropayments increases or decreases industry profit and consumer surplus. When all websites are highly effective at converting ads into revenue, the introduction of micropayments benefits all parties: in the new equilibrium, websites place more ads but reward consumers for watching ads through monetary transfers. In other cases, micropayments may benefit only websites by enabling them to charge consumers for the content and thus extract surplus.

While we motivate the model as one in which websites place ads, we can alternatively interpret the strategy of a website as the amount of personal data it collects from visiting users.

For example, collecting personal data imposes disutility on visitors, and websites may differ in their ability to monetize data. Thus our model also speaks to coordination problems of websites that request access to data from visitors.

Overall, we indentify ways in which complementary actors in the attention economy, such as websites and browsers, may fail to coordinate their strategies to place ads or collect data, and we study potential remedies for this failure. When monetizing attention or data imposes disutility on consumers, an equilibrium entails both classical and novel market distortions–i.e., double marginalization and misplacement. Competition or monetary instruments play different and interesting roles in correcting or exacerbating such distortions.

Related Literature Double marginalization, first pointed out by Cournot (1838), has been extensively studied in the context of complementary goods (e.g., Spengler (1950); Rey and Tirole (1986); Shleifer and Vishny (1993); Lerner and Tirole (2004, 2015)). In contrast, to the best of our knowledge, misplacement, which leads to our central trade-off, has not been discussed in the extant literature, especially in the setup where multiple firms can demand attention from consumers trying to accomplish a single task. The closest analog to misplacement of which we are aware arises in Schwartz (1989), where imperfectly competing sellers of substitutes have asymmetric marginal costs. Dellarocas's (2012) model resembles ours in that it also studies double marginalization in online advertising. However, our paper differs from his model in that he studies the double markup problem in product pricing, whereas we focus on the interaction between ads misplacement and the double marginalization in the amount of advertisement.

A strand of literature studies "vertical cooperative advertising." It stems from Berger (1972), continues through Cao and Ke (2019), and is surveyed by Jørgensen and Zaccour (2014). There, coordination problems arise between manufacturers and retailers, each of which may place ads to increase demand. In contrast, our paper focuses on obstacles faced by multiple websites (or, more broadly, ad-funded platforms) in coordinating policies determining how much advertisement to show.

Another strand of literature studies situations in which platforms, such as search engines, direct users to sellers (e.g., Hagiu and Jullien (2011, 2014); Eliaz and Spiegler (2011); White

(2013); Gomes (2014); Burguet et al. (2015); de Cornière (2016)). These models do not have the misplacement that we study, because the sellers can make monetary transfers to the platform. Our model relates to de Cornière and Taylor (2014), because in both models users surf from an organizer to content producers, which cannot make monetary transfers to one another. Suppose that multiple content producers compete in one category of content. Then, if we introduce horizontal differentiation across websites, the organizer may be biased against the websites that display many ads. de Cornière and Taylor (2014) study such a recommendation bias in selecting one group of publishers rather than the other.[2] In their terminology, we abstract away from the recommendation bias and study the lack of coordination between the search engine and publishers, particularly the roles of competition and payments.

2 Baseline Model

The model consists of a unit mass of consumers and two websites, 1 and 2. The game has two stages. First, each website $i \in \{1, 2\}$ simultaneously chooses the amount $a_i \in \mathbb{R}_+$ of ads to place, which is publicly observable. Second, each consumer learns her value of visiting websites, then decides which websites to visit.

The payoff to each consumer is the gross benefit of visiting websites minus the disutility from ads. The websites are perfectly complementary: if a consumer visits both websites, she receives a gross benefit of v; otherwise, she obtains a benefit of zero. Visiting a single website is never optimal, so without loss of generality, we can assume that consumers choose between visiting both websites and visiting none. The benefit v is drawn from cumulative distribution function F that has a positive continuous density f on its support $[0, \overline{v}]$ with $\overline{v} > 0$ and has an increasing hazard rate $\frac{f}{1-F}$.[3] For each website i they visit, consumers incur a disutility of $\delta_i(a_i)$. For each i, $\delta_i(\cdot) : \mathbb{R}_+ \to \mathbb{R}_+$ is strictly increasing, strictly convex, and continuously differentiable, and satisfies $\delta_i'(0) = 0$. To sum up, if a consumer visits both websites, her payoff

[2] Similarly, Burguet, Caminal, and Ellman (2015) consider the situation in which search ads and display ads are substitutes and study the search engine's incentives to distort search results.

[3] If the support of F is $[\underline{v}, \overline{v}]$ for some $\underline{v} > 0$, there could be an equilibrium in which all consumers visit websites and thus we cannot use the first-order condition for a website's optimization. To simplify the analysis, we assume $\underline{v} = 0$.

is $v - \delta_1(a_1) - \delta_2(a_2)$. Otherwise, her payoff is zero.

The payoff of each website is its advertising revenue. Specifically, website i earns a payoff of $a_i \cdot m$, where m is the mass of consumers who visit website i. When consumers behave optimally, we have $m = 1 - F(\delta_1(a_1) + \delta_2(a_2))$.

Our solution concept (hereafter, "equilibrium") is pure-strategy subgame perfect equilibrium (SPE) in which a positive mass of consumers visit websites. The restriction excludes a trivial SPE in which both websites set a large a_i and no one visits websites (Appendix A proves the existence of equilibrium).

2.1 Discussion of Modeling Assumptions

Heterogeneous websites. One of our key insights (i.e., misplacement) is relevant when consumers face a higher marginal disutility from advertising on one website than the other, e.g., $\delta_1'(a) > \delta_2'(a)$ for all $a > 0$. For example, suppose that a consumer browses a social media website (website 1) and a news website (website 2) to learn about some event. The two websites are complements, e.g., the social media offers a timely but brief description of the event, and the news website offers a detailed description based on videos. Suppose the former uses display ads and the latter uses video ads. If consumers find video ads that appear prior to news videos more annoying than display ads, they face higher δ_2' than δ_1'.[4] Generally, such a difference between $\delta_1(\cdot)$ and $\delta_2(\cdot)$ would arise if websites employ different advertising modes, and consumers find one mode more annoying than the other.

The baseline model assumes that websites earn the same revenue per unit of advertisement. Section 6 studies a general model in which each website i earns a revenue of $r_i(a_i)$ per visit. We show that a lower marginal disutility $\delta_i'(\cdot)$ is equivalent to a higher marginal revenue $r_i'(a_i)$ from placing ads. For example, suppose website 1 has better access to user data, enabling advertisers to target users; consequently, it can sell advertising slots at higher prices. Website 1 then faces a higher marginal revenue from placing ads than website 2, which is equivalent

[4]Academic empirical evidence on consumers' attitudes toward different advertising modes is sparse, but there is more suggestive evidence of this example from non-academic media sources, e.g., https://www.emarketer.com/content/why-consumers-avoid-ads and https://www.vieodesign.com/blog/new-data-why-people-hate-ads.

to $\delta_1'(a) < \delta_2'(a)$. Generally, given the equivalence we show in Section 6, δ_i' will be higher than δ_j' when website i is better than website j at converting ads into revenue, because of, e.g., heterogeneous demand from advertisers or access to data in the unmodeled advertising market.

Disutility function. The baseline model assumes that the total disutility is the sum of disutilities from two websites. Section 6 shows that the main insight continues to hold in a general setting where consumers incur a disutility of $C(\delta_1(a_1) + \delta_2(a_2))$ for some increasing convex function $C(\cdot)$.

Complementarity of websites. We assume that the two websites are perfect complements. Appendix B relaxes this assumption and studies a model in which some fraction of consumers do not face such complementarity. We establish the robustness of our main results and comparative statics with respect to the degree of complementarity, which is captured by the fraction of consumers who regard two websites as complements.

Advertising market. The baseline model does not explicitly model advertisers. Appendix C microfounds the revenue of each website in line with Anderson and Coate (2005), where advertisers explicitly demand advertising slots, and websites act as a quantity-setting monopolist in the respective advertising markets.

Zero marginal cost. We assume that websites face zero marginal cost of serving users. We impose such an assumption for two reasons. First, the near-zero marginal cost of serving users is a feature of digital goods and online services (Rifkin, 2014). Second, while some of our results depend on the assumption of zero marginal cost, introducing a positive cost (i.e., each website earns $a_i - c_i$ per visit) increases the notational burden for other results without adding new insights.[5] Thus we assume $c_1 = c_2 = 0$.

[5] Specifically, Propositions 1 and 3 continue to hold when $c_i > 0$ for each $i = 1, 2$. Propositions 2 and 4 may not hold, but we have the same insights for a small positive c_i (which is reasonable for digital goods) because the relevant equilibrium objects are continuous in (c_1, c_2). For example, in the setting of Proposition 2, the difference between the total disutilities at the equilibrium and the total disutilities at the industry optimum is arbitrarily small when $\max_i c_i$ approaches zero.

2.2 Alternative Interpretation: Data Collection and Privacy Loss

We describe the model, primarily, as one of online advertising markets, where a_i captures a website's advertising volume. Alternatively, we can interpret the model in the context of data collection and privacy loss. For example, suppose firm 1 is now the maker of a smartphone browser, and firm 2 is a website. In this example, the browser and the website are complementary components that fulfill consumers' needs to access content. Leaving aside any issues of advertisement to the primary interpretation, in this context the focus is on the collection that each firm does of users' personal data. We can now interpret a_i as the level of data collection and $\delta_i(a_i)$ as the disutility from privacy loss. Collecting more data imposes a higher disutility on consumers but increases the revenue per consumer of the browser or the website, possibly because of better targeting.

The browser and website may differ in the disutility they impose on consumers or, in view of the generalization presented in Section 6, in the value they extract from user data. For example, both the browser and the website may collect data in a way that purports to improve the user experience, but the browser may store this more securely than the website and thus impose less perceived privacy loss. On the revenue side, if one of the two firms also operates a large-scale data-driven business with other components, it potentially can earn more from a given amount of data that it collects. In view of this interpretation, the coordination problems we study are not purely limited to situations that involve advertising but also can be seen to arise in other online settings with complementary components in which monetary transfers are not practical.

3 Equilibrium

We identify two forces that render equilibrium suboptimal in terms of industry profit and social welfare. We say that $(a_1^\Pi, a_2^\Pi) \in \mathbb{R}_+^2$ is *industry-optimal* if it maximizes the joint profit given consumers' optimal behavior:

$$(a_1^\Pi, a_2^\Pi) \in \arg\max_{(a_1,a_2)\in\mathbb{R}_+^2} (a_1 + a_2)\left[1 - F\left(\delta_1(a_1) + \delta_2(a_2)\right)\right]. \tag{1}$$

Definition 1. An equilibrium entails *double marginalization* if the total disutility $\delta_1(a_2) + \delta_2(a_2)$ at the equilibrium is strictly greater than the one at the industry optimum.

Definition 2. An equilibrium entails *misplacement* if the websites can jointly deviate and strictly increase their total profits while keeping total disutility at the equilibrium level.

Proposition 1. *Take any equilibrium, and let a_i^* denote the equilibrium strategy of each website i. Then the following holds.*

1. *The equilibrium entails double marginalization.*

2. *The equilibrium entails misplacement if $\delta_1'(a_1^*) \neq \delta_2'(a_2^*)$. In particular, any equilibrium entails misplacement if $\delta_i'(a) > \delta_j'(a)$ for all $a > 0$, $i \neq j$.*

Proof. Consumers visit websites if and only if $v \geq \delta_1(a_1) + \delta_2(a_2)$. In equilibrium, each website $i \in \{1, 2\}$ chooses a_i to maximize $a_i D(\delta_i(a_i) + \delta_j(a_j))$ given the other website j's choice a_j, where $D(\cdot) = 1 - F(\cdot)$.

For Point 1, note that a_i^Π, $i = 1, 2$, solve the first-order condition for the industry-profit maximization:

$$\begin{aligned} &1 - F(\delta_1(a_1^\Pi) + \delta_2(a_2^\Pi)) - (a_1^\Pi + a_2^\Pi) f(\delta_1(a_1^\Pi) + \delta_2(a_2^\Pi))\delta_i'(a_i^\Pi) = 0 \\ \Rightarrow\ & a_1^\Pi + a_2^\Pi = \frac{g(\delta_i(a_1^\Pi) + \delta_j(a_2^\Pi))}{\delta_i'(a_i^\Pi)}, \end{aligned} \tag{2}$$

where $g(\cdot) = \frac{1-F(\cdot)}{f(\cdot)}$ is decreasing. The equilibrium strategy a_i^* of each website i solves the first-order condition[6]

$$\begin{aligned} &1 - F(\delta_1(a_1^*) + \delta_2(a_2^*)) - a_i^* f(\delta_1(a_1^*) + \delta_2(a_2^*))\delta_i'(a_i^*) = 0 \\ \Rightarrow\ & a_i^* = \frac{g(\delta_i(a_i^*) + \delta_j(a_j^*))}{\delta_i'(a_i^*)} \\ \Rightarrow\ & a_1^* + a_2^* = g(\delta_1(a_1^*) + \delta_2(a_2^*)) \sum_i \frac{1}{\delta_i'(a_i^*)}. \end{aligned} \tag{3}$$

[6] Unless explicitly stated otherwise, all summations $\sum_i$ in the paper mean $\sum_{i \in \{1,2\}}$.

Suppose to the contrary that $\sum_i \delta_i(a_i^\Pi) \geq \sum_i \delta_i(a_i^*)$. This implies $D\left(\sum_i \delta_i(a_i^\Pi)\right) \leq D\left(\sum_i \delta_i(a_i^*)\right)$, and thus $\sum_i a_i^\Pi \geq \sum_i a_i^*$, since, if $\sum_i a_i^\Pi < \sum_i a_i^*$, the industry profit would be strictly greater under (a_1^*, a_2^*) than (a_1^Π, a_2^Π). Combining these inequalities and (3), we obtain

$$\sum_i a_i^\Pi \geq g\left(\sum_i \delta_i(a_i^\Pi)\right) \sum_i \frac{1}{\delta_i'(a_i^*)} > \frac{g\left(\sum_i \delta_i(a_i^\Pi)\right)}{\delta_i'(a_i^\Pi)},$$

which contradicts (2). The last inequality holds because $\sum_i \delta_i(a_i^\Pi) \geq \sum_i \delta(a_i^*)$ implies that there is some i such that $a_i^\Pi \geq a_i^*$, so $\delta_i'(a_i^\Pi) \geq \delta_i'(a_i^*)$.

We now show Point 2. Misplacement occurs if $\delta_1'(a_1^*) \neq \delta_2'(a_2^*)$, because the websites can adjust (a_1, a_2) to increase the revenue per visit, $a_1 + a_2$, while satisfying $\delta_1(a_1) + \delta_2(a_2) = \delta_1(a_1^*) + \delta_2(a_2^*)$. Suppose $\delta_1'(a) > \delta_2'(a)$ for all $a > 0$ but $\delta_1'(a_1^*) = \delta_2'(a_2^*)$ in equilibrium. The first-order conditions in equilibrium imply $a_1^* = a_2^*$, which contradicts $\delta_1'(a_1^*) > \delta_2'(a_2^*)$. $\square$

The intuition of Point 1 is as follows. When the existing users view one more ad on a given website, this firm earns additional revenue. However, this firm's placing another ad decreases the number of users who visit both websites. In equilibrium, websites fail to internalize this negative effect on each other and place more ads than the industry optimum.

Point 2 says that an equilibrium typically entails misplacement when websites have different disutility functions. To see the intuition in the starkest way possible, suppose that websites 1 and 2 impose disutilities of 1 and 2 per unit of ads, respectively.[7] For example, website 2 may embed ads in a video, which consumers find more distracting. In equilibrium, website 2 chooses some positive number of ads to maximize its profit. However, websites could increase their joint profit without changing the total advertising volume if website 1 alone placed ads ($a_1 = a_1^* + a_2^*$), because doing so minimizes consumer disutility and maximizes their visits, given the total ad volume.

We define double marginalization and misplacement in terms of the industry profit. However, the results have implications for efficiency because any outcome with either of these

[7] For maximal simplicity, this example uses linear disutility functions with differing slopes. Thus, technically, it is outside the scope of our model, which assumes these functions to be strictly convex.

properties is Pareto dominated.[8] For example, in an equilibrium with misplacement, websites can increase the joint profit without changing total disutility, which also implies that they can strictly increase the joint profit and consumer surplus by adjusting (a_1, a_2). Note that double marginalization or misplacement alone implies that the equilibrium is inefficient. Thus the equilibrium continues to be inefficient after changing the market structure or the websites' business models, unless the change eliminates both distortions. The observation is relevant to the analysis in the following sections.

How to alleviate these distortions? We begin with a solution for double marginalization. When a final good comprises complementary components, a well-known solution to the double marginalization problem is introducing competition in the markets for all but one of the individual components. For instance, suppose "hardware" and "software" are two perfectly complementary products and produced by different firms. The firms face double marginalization, but if the hardware market became perfectly competitive, the software maker would be able to charge a price that implements the outcome that maximizes the industry profit.[9] Would the same logic apply to our setting?

4 Competition

Suppose now that website 2 faces perfect competition. Formally, the game with competition consists of websites 1, 2, and $2'$. Websites 2 and $2'$ have the same disutility function δ_2, and consumers earn gross benefit v if and only if they visit website 1 and at least one of websites 2 and $2'$. As in the baseline model, consumers incur disutility $\delta_i(a_i)$ for each website they visit.

[8] Here, efficiency is from the view of consumers and websites. Alternatively, we may include advertisers' surplus as part of the efficiency criterion. In such a case, the same argument holds if websites extract full surplus from advertisers. Otherwise, the direction of inefficiency may depend on how we model the advertising market—e.g. websites may place too few ads if they do not internalize advertisers' surplus. In contrast, the model already captures the value of ads to consumers in a reduced-form way, because δ_i can include the consumer's surplus from potential transactions with advertisers.

[9] Casadesus-Masanell, Nalebuff, and Yoffie (2007) and Cheng and Nahm (2007) study variations of such models for hardware and software. They study the case in which the producers in the sectors with competition are vertically differentiated. Some papers study the use of competition among firms in a particular category of a complementary bundle as a solution to the double marginalization problem. Dellarocas (2012) studies a related idea with performance-based fees in online advertising. More broadly, see, e.g., Rey and Tirole (1986), Shleifer and Vishny (1993), and Lerner and Tirole (2004, 2015).

Thus consumers choose among visiting: no website; websites 1 and 2; and websites 1 and $2'$.[10]

The standard logic of Bertrand competition implies that $a_2 = a_{2'} = 0$ in equilibrium.[11] The following result, which summarizes the impact of competition, uses an assumption on the functional form of disutility functions.

Assumption 1. For each $i \in \{1, 2\}$, $\delta_i(a) = \gamma_i a^k$ for some $\gamma_i > 0$ and $k > 1$.

Proposition 2. *Under Assumption 1, the following holds.*

1. *Competition eliminates double marginalization—i.e., the equilibrium total disutility under competition equals the one under the industry-optimum. Thus competition increases consumer surplus.*

2. *Compared to the baseline model, competition increases the industry profit if and only if $\gamma_2 \geq \gamma^*$ for some $\gamma^* > 0$. The profit comparison is strict whenever $\gamma_2 \neq \gamma^*$.*

Proof. We show Point 1. Without competition, the equilibrium condition for website $i \in \{1, 2\}$ is

$$\begin{aligned} a_i^* &= \frac{g(\delta_1(a_1^*) + \delta_2(a_2^*))}{\delta_i'(a_i^*)} \\ \Rightarrow \ \delta_i(a_i^*) &= \frac{g(\delta_1(a_1^*) + \delta_2(a_2^*))}{k} \quad (4) \\ \Rightarrow \ \delta_1(a_1^*) + \delta_2(a_2^*) &= \frac{2}{k} g(\delta_1(a_1^*) + \delta_2(a_2^*)). \quad (5) \end{aligned}$$

The second equality uses $\delta_i(a) = \gamma_i a^k$, which implies $a\delta_i'(a) = k\delta_i(a)$. Under competition, the equilibrium choice a_1^C of website 1 solves

$$a_1^C = \frac{g(\delta_1(a_1^C))}{\delta_i'(a_1^C)} \Rightarrow \delta_1(a_1^C) = \frac{1}{k} g(\delta_1(a_1^C)). \quad (6)$$

[10] As in the baseline model, we could allow consumers to visit only one website or all websites, but doing so will never be a part of a consumer's equilibrium strategy.

[11] Recall that we focus on equilibrium in which a positive mass of consumers visit websites. Thus if $a_2 > 0$ and $a_{2'} > 0$ in equilibrium, a website with a weakly higher a_i, say website 2, can undercut $a_{2'}$. If $a_2 > a_{2'} = 0$ or $a_{2'} > a_2 = 0$, website $i \in \{2, 2'\}$ with $a_i = 0$ can profitably deviate by slightly increasing a_i. Therefore we have $a_2 = a_{2'} = 0$ in any equilibrium.

Comparing the two equilibrium conditions, we obtain $\delta_1(a_1^*) + \delta_2(a_2^*) > \delta_1(a_1^C)$. Thus competition increases consumer surplus. Under the industry-optimum, we have $\delta_1'(a_1^\Pi) = \delta_2'(a_2^\Pi)$, which implies $a_2^\Pi \delta_1'(a_1^\Pi) = a_2^\Pi \delta_2'(a_2^\Pi) = k\delta_2(a_2^\Pi)$. Thus we obtain

$$a_1^\Pi + a_2^\Pi = \frac{g(\delta_i(a_1^\Pi) + \delta_j(a_2^\Pi))}{\delta_1'(a_1^\Pi)} \Rightarrow \delta_1(a_1^\Pi) + \delta_2(a_2^\Pi) = \frac{1}{k} g(\delta_i(a_1^\Pi) + \delta_j(a_2^\Pi)). \tag{7}$$

Equations (6) and (7) imply that $\delta_1(a_1^C) = \delta_i(a_1^\Pi) + \delta_j(a_2^\Pi)$, that is, the equilibrium under competition and the industry optimum have the same total disutility.

To show Point 2, we show that the industry profit without competition decreases in γ_2. Equation (5) implies that the total equilibrium disutility, which we denote by Δ^*, is independent of (γ_1, γ_2). Equation (4) implies that the equilibrium strategy a_i^* of website i solves $(a_i^*)^k = \frac{g(\Delta^*)}{k\gamma_i}$. Thus a higher γ_2 decreases a_2^* without changing a_1^* and the total disutility. As a result, the industry profit at equilibrium without competition is decreasing in γ_2. As $\gamma_2 \to 0$, the industry profit diverges to ∞. As $\gamma_2 \to \infty$, the industry profit at equilibrium is $a_1^*[1 - F(\Delta^*)]$ without competition and $a_1^C[1 - F(\delta_1(a_1^C))]$ under competition. Comparing (5) and (6), we obtain $\Delta^* > \delta_1(a_1^C)$. Under competition, website 1 can choose a_1 to satisfy $\delta_1(a_1) = \Delta^*$ and secure a payoff of $a_1[1 - F(\Delta^*)] > a_1^*[1 - F(\Delta^*)]$. (By (4), we obtain $\delta_1(a_1^*) = \Delta^*/2$, so $a_1 > a_1^*$). Thus we have $a_1^*[1 - F(\Delta^*)] < a_1^C[1 - F(\delta_1(a_1^C))]$. Therefore, there is a unique positive threshold γ^* such that the industry profit is greater under competition if and only if $\gamma_2 \geq \gamma^*$. □

Proposition 2 confirms that competition eliminates double marginalization: Competition forces website 2 to set $a_2 = 0$ and enables website 1 to act as a monopolist. Under Assumption 1, the problem of website 1 to choose a total disuility coincides with that of the industry optimum. Moreover, competition benefits consumers by reducing total disutility.

However, Proposition 2 shows that we must be cautious in applying the aforementioned conventional wisdom to our model. In particular, competition could exacerbate misplacement and reduce the industry profit. To see this, consider a simple variant of our setup with linear disutility from advertisement. Suppose websites 1 and 2 impose disutilities of 1 and d per unit of ads, respectively. If $d > 1$, only website 1 should place ads at the industry optimum. Such an outcome arises if website 2 faces competition and is forced to set $a_2 = 0$. However, if $d < 1$, competition for website 2 exacerbates misplacement: Only website 2 should place ads at the industry optimum, but competition decreases the number of ads on website 2 and increases the number on website 1. The resulting change increases consumer visits, but the total advertising volume decreases significantly, because the increase in ads on website 1 does not compensate for the decrease in ads on website 2. Thus, competition for the efficient segment of the market could reduce the industry profit. Point 2 of the proposition formalizes this intuition in terms of the disutility parameter, γ_2, of website 2.

Finally, as $\gamma_2 \to 0$, industry profit in the absence of competition goes to ∞, so the reduction of industry profit due to competition goes to ∞. Thus, for a sufficiently small γ_2, perfect competition for website 2 could decrease total surplus, i.e., the sum of consumer and website payoffs.

5 Micropayments

Recall from Proposition 1 that an equilibrium entails misplacement if $\delta_1'(a_1^*) \neq \delta_2'(a_2^*)$. In such a case, the websites can jointly adjust (a_1, a_2) to increase the industry profit per visit (i.e., $a_1 + a_2$) while keeping the total disutility at $\delta_1(a_1) + \delta_2(a_2) = \delta_1(a_1^*) + \delta_2(a_2^*)$. Can websites eliminate misplacement without such explicit coordination? One way is that websites use another strategic variable to equalize their equilibrium marginal disutilities. In this spirit, we extend the model by allowing websites to not only place ads and but also to charge or subsidize consumers using per-visit *micropayments*.[12] To capture this idea, we focus on the case in which all websites can use micropayments. Appendix D studies an alternative case in which only one website can use micropayments.

Formally, we extend the baseline model as follows. First, each website i simultaneously chooses the number a_i of ads and price $t_i \in \mathbb{R}$. Second, each consumer observes $(a_i, t_i)_{i=1,2}$, then decides which websites to visit. The payoff to each consumer is $v - \delta_1(a_1) - \delta_2(a_2) - t_1 - t_2$ if she visits both websites, and zero otherwise. The payoff to website i is $(a_i + t_i) \cdot m$, where

[12]The micropayment scenario we consider differs from a subscription business model, in which users pay, for instance, a monthly fee in exchange for "all you can eat" access to a website. Instead, micropayments should affect each momentary decision of attention allocation, in a manner embodied, for instance, by the Basic Attention Token (BAT) offered by the Brave web browser.

m is the mass of visiting consumers.

The following notions are useful for describing results. We use $\Delta_i(a_i, t_i)$ (or simply, Δ_i) for $\delta_i(a_i) + t_i$, which is the total disutility—including the nuisance from ads and monetary transfer—that consumers incur from visiting website i. In the model with micropayments, we define double marginalization and misplacement in terms of Δ_i. Specifically, an equilibrium entails double marginalization if the equilibrium total disutility $\Delta_1 + \Delta_2$ is strictly greater than the one at the industry optimum.[13] An equilibrium entails misplacement if the websites can change (a_1, t_1, a_2, t_2) to increase the industry profit while keeping $\Delta_1 + \Delta_2$ at the equilibrium level.

5.1 No Misplacement with Micropayments

Micropayments eliminate misplacement but not double marginalization.

Proposition 3. *In the game with micropayments, any equilibrium entails double marginalization but does not entail misplacement.*

Proof. We characterize the equilibrium total disutility, $\Delta_1^* + \Delta_2^*$. Given Δ_j^*, website i solves

$$\max_{(a_i,t_i)} (a_i + t_i)D(\delta_i(a_i) + t_i + \Delta_j^*). \tag{8}$$

The first-order conditions with respect to a_i and t_i imply $\delta_i'(a_i^*) = 1$. Thus, if website i chooses total disutility Δ_i, the optimal t_i is $t_i = \Delta_i - \delta_i(a_i^*)$. Plugging this into the above problem, we obtain

$$\max_{\Delta_i \geq 0} (\Delta_i + a_i^* - \delta_i(a_i^*))D(\Delta_i + \Delta_j^*). \tag{9}$$

The first-order condition with respect to Δ_i is

$$\Delta_i^* + a_i^* - \delta_i(a_i^*) = g(\Delta_i^* + \Delta_j^*), \tag{10}$$

[13] The total disutility at the industry optimum is now $\Delta_1(a_1^\Pi, t_1^\Pi) + \Delta_2(a_2^\Pi, t_2^\Pi)$, where $(a_1^\Pi, t_1^\Pi, a_2^\Pi, t_2^\Pi) \in \arg \max_{(a_1,t_1,a_2,t_2)} (a_1 + t_1 + a_2 + t_2)D(\delta_1(a_1) + t_1 + \delta_2(a_2) + t_2)$.

which implies

$$\sum_i \Delta_i^* + \sum_i [a_i^* - \delta_i(a_i^*)] = 2g\left(\sum_i \Delta_i^*\right). \tag{11}$$

Similarly, the industry optimum, $(\Delta_1^\Pi, \Delta_2^\Pi)$, satisfies

$$\sum_i \Delta_i^\Pi + \sum_i [a_i^\Pi - \delta_i(a_i^\Pi)] = g\left(\sum_i \Delta_i^\Pi\right). \tag{12}$$

Comparing the two conditions, we obtain $\sum_i \Delta_i^* > \sum_i \Delta_i^\Pi$, i.e., any equilibrium entails double marginalization.

To show there is no misplacement, suppose that websites jointly maximize the industry profit subject to the constraint that the total disutility is $\sum_i \Delta_i^*$, i.e., they solve

$$\begin{aligned} &\max_{(a_1,t_1,a_2,t_2)} \; a_1 + t_1 + a_2 + t_2 \\ &s.t. \quad \delta_1(a_1) + t_1 + \delta_2(a_2) + t_2 = \sum_i \Delta_i^*. \end{aligned}$$

Solving this problem, we obtain $a_1 = a_1^*$, $a_2 = a_2^*$, and $t_1 + t_2 = \sum_i \Delta_i^* - \sum_i \delta_i(a_i^*)$. Because the equilibrium outcome satisfies the above condition, the websites cannot increase total profits while keeping the total disutility constant. Thus, no equilibrium entails misplacement. □

The result of no misplacement under micropayment is intuitive. Suppose that in equilibrium, website i chooses a_i such that $\delta_i'(a_i) < 1$. Then it could profitably deviate by increasing the advertising level a_i and decreasing t_i, while holding fixed total user disutility. Similarly, if $\delta_i'(a_i) > 1$, website i will have a profitable deviation.[14] As a result, the equilibrium with micropayment satisfies $\delta_1'(a_1) = \delta_2'(a_2) = 1$. The marginal disutility of advertising equals the marginal disutility of money across all websites, and thus the equilibrium entails no misplacement. The result implies that misplacement crucially depends on the lack of another competitive instrument, a price for the service. To put it differently, the ad-financed revenue model drives misplacement. In contrast, micropayment does not eliminate double marginalization

[14] A similar finding appears in the literature studying free-to-air versus subscription television (Choi, 2006; Crampes, Haritchabalet, and Jullien, 2009; Peitz and Valletti, 2008).

because websites do not internalize the effect of increasing a_i or t_i on the others' profits.

5.2 Welfare Effects of Micropayments

The following result and subsequent discussion show that the introduction of micropayments can increase or decrease consumer surplus and industry profits.

Proposition 4. *Under Assumption 1, consumer surplus is greater in the game with micropayments than in the baseline model if and only if* $\sum_i \left(\frac{1}{\gamma_i}\right)^{\frac{1}{k-1}}$ *is above some threshold. Industry profit is greater in the game with micropayments if* $\min(\gamma_1, \gamma_2)$ *is sufficiently large.*

Proof. Under Assumption 1, the proof of Proposition 2 shows that without micropayments, consumer surplus is independent of (γ_1, γ_2). With micropayments, the equilibrium choice of a_i^* satisfies $\delta_i'(a_i^*) = 1$, or equivalently, $a_i^* = \left(\frac{1}{k\gamma_i}\right)^{\frac{1}{k-1}}$. Equation (11) becomes

$$\sum_i \Delta_i^* + \left(1 - \frac{1}{k}\right)\left(\frac{1}{k}\right)^{\frac{1}{k-1}} \sum_i \left(\frac{1}{\gamma_i}\right)^{\frac{1}{k-1}} = 2g\left(\sum_i \Delta_i^*\right). \tag{13}$$

This equation implies that the equilibrium total disutility with micropayments is decreasing in $\sum_i \left(\frac{1}{\gamma_i}\right)^{\frac{1}{k-1}}$, which completes the proof of the first part.

To show the second part, let q satisfy $q - 2g(q) = 0$. Equation (13) implies that as $\min(\gamma_1, \gamma_2) \to \infty$, we have $\Delta_1^* + \Delta_2^* \to q$ and $(a_1^*, a_2^*) \to (0, 0)$, so the industry profit converges to $qD(q) > 0$. Without micropayments, the industry profit converges to 0 as $\min(\gamma_1, \gamma_2) \to \infty$, because the total disutility is constant while the number of ads websites place approaches zero. $\square$

Intuitively, websites benefit from micropayments when their advertising technology is inefficient (i.e., γ_i is large). In such cases, micropayments allow them to reduce the advertising volume and charge consumers for access. By doing so, websites can extract more surplus from consumers.

Proposition 4 highlights the potential benefit of micropayments for the industry. However, it is not the case that micropayments always increase total profits. Appendix D provides a numerical example in which the introduction of micropayments decreases industry profits and consumer surplus. There, the websites are symmetric and the equilibrium without micropayments does not entail misplacement. The introduction of micropayments reduces the equilibrium advertising volume, but each website also charges a positive price to consumers, which leads to a higher total disutility than without micropayments. In this example, micropayments exacerbate double marginalization and hurt both consumers and websites.

A natural question is how Propositions 3 and 4 would change if only a subset of websites adopt micropayments. The result that micropayments eliminate misplacement depends crucially on all websites adopting them. Equilibrium can still entail misplacement when only one website adopts micropayments.[15] Indeed, Appendix D shows that the adoption of micropayments by only one website could exacerbate misplacement—e.g., in a symmetric environment with $\delta_1(\cdot) = \delta_2(\cdot)$, the game without micropayments has no misplacement, but the game in which only one website can use micropayments typically entails misplacement.

Appendix D also shows that if the website that can use micropayments has an efficient advertising technology (i.e., a small γ_i), consumer surplus and the payoff to the other website increase, compared to the case with no micropayments.

6 General Payoff Specification

The baseline model assumes that two websites are perfect complements and earn the same revenue per unit of ads. Also, there, a consumer's payoff is additively separable with respect to each δ_i. We now relax these assumptions and extend the previous results. This generalization allows us to decompose the possible sources of misplacement and, in doing so, highlights the potential pervasiveness of this phenomenon.

6.1 General Model Without Micropayments

To clarify the connection between the baseline model and the general model introduced here, we write the strategy of website i as $\alpha_i \geq 0$, which represents the number of ads (or, under the alternative interpretation of Section 2.2, the level of data collection). Website i earns a

[15] See Example 2 in Appendix D.

revenue of $r_i(\alpha_i)$ per visit, where $r_i : \mathbb{R}_+ \to \mathbb{R}_+$ is strictly increasing, weakly concave, and continuously differentiable, and satisfies $r_i(0) = 0$. If a consumer visits both websites, she now earns a payoff of $v - C(\delta_1(\alpha_1) + \delta_2(\alpha_2))$, where v is drawn from the cumulative distribution function F that satisfies the same assumption as in Section 2. Here, $C : \mathbb{R}_+ \to \mathbb{R}_+$ is strictly increasing, weakly convex, and continuously differentiable. To exclude equilibria with $\alpha_i = 0$, we assume either (i) each δ_i is strictly convex and $\lim_{\alpha \to 0} \delta_i'(\alpha) = 0$, or (ii) each δ_i is weakly convex, and each r_i is strictly concave and satisfies $\lim_{\alpha \to 0} r_i'(\alpha) = \infty$.[16] For example, we may assume that disutility takes the form of $C(\alpha_1 + \alpha_2)$, where consumers view ads across the two sites to be completely fungible, in terms of the disutility they create.

Websites can now be heterogeneous in $r_i(\alpha_i)$, which maps the advertising volume or the level of data collection into revenue. Heterogeneity may come from websites' different abilities to target users or monetize data. Appendix C explicitly models the advertising market in line with Anderson and Coate (2005) and motivates the heterogeneity of $r_i(\cdot)$ functions by modeling advertisers' differing demands for two websites' advertising slots.

6.2 Equivalence

In this section, Model 1 refers to our baseline model in Section 3, and Model 2 refers to the above general model. We show that Model 2 is equivalent to Model 1 in which consumers draw v according to $F(C(\cdot))$, and website i has a disutility function of $\delta_i(r_i^{-1}(a))$.

To show the equivalence, take any (r_1, r_2, C, F) that satisfies the assumptions of the previous section. In Model 2, consumers visit websites if $v \geq C(\delta_1(\alpha_1) + \delta_2(\alpha_2))$, which occurs with probability $1 - F(C(\delta_1(\alpha_1) + \delta_2(\alpha_2))$. In equilibrium, given the strategy α_j of website j, website i solves

$$\begin{aligned} &\max_{\alpha \geq 0} \quad r_i(\alpha)\,[1 - F(C(\delta_i(\alpha) + \delta_j(\alpha_j))] \\ =&\max_{a \geq 0} \quad a\left[1 - F(C(\delta_i(r_i^{-1}(a)) + \delta_j(r_j^{-1}(a_j)))\right], \end{aligned} \tag{14}$$

[16] Recall that the baseline model assumes $r_i(a) = a$ and requires δ_i to be strictly convex. In the current setup, if δ_i is only weakly convex (e.g., linear δ_i), the conditions on r_i ensure that the transformed disutility function $\delta_i(r_i^{-1}(a))$ we use below is strictly convex and satisfies $\lim_{a \to 0} \delta_i(r_i^{-1}(a))$=0.

where $a_j = r_j(\alpha_j)$. The above equality holds because we can write website i's strategy as $a := r_i(\alpha)$ instead of α_i. The problem in (14) is website i's problem in Model 1 in which v is drawn from $F(C(\cdot))$ and the disutility function of website i is $\delta_i(r_i^{-1}(\cdot))$.

The assumptions we impose on Model 2 ensure that $F(C(\cdot))$ and each $\delta_i(r^{-1}(\cdot))$ satisfy the assumptions for Model 1. For example, because F has an increasing hazard rate and C is convex, the hazard rate of $F(C(\cdot))$, which is $\frac{f(C(\cdot))\cdot C'(\cdot)}{1-F(C\cdot))}$, is also increasing. As a result, (α_1^*, α_2^*) is an equilibrium outcome of Model 2 if and only if $(a_1^*, a_2^*) := (r_1(\alpha_1^*), r_2(\alpha_2^*))$ is an equilibrium outcome of Model 1 with value distribution $F(C(\cdot))$ and disutility function $\delta_i(r^{-1}(\cdot))$.

This generalization provides a finer-grained description of the sources of misplacement. In Model 1, Proposition 1 states that the equilibrium entails misplacement if $\delta_i'(a) > \delta_j'(a)$ for all a. In Model 2, the condition is equivalent to $\frac{d}{da}\delta_i(r_i^{-1}(a)) > \frac{d}{da}\delta_j(r_j^{-1}(a))$, which is $\frac{\delta_i'(\alpha)}{r_i'(\alpha)} > \frac{\delta_j'(\alpha)}{r_j'(\alpha)}$ for all α. As a result, misplacement arises even if, in equilibrium, $\delta_1'(\alpha_1) = \delta_2'(\alpha_2)$, so long as one website has a greater marginal revenue, $r_i'(\cdot)$, of placing ads than the other website. This is particularly relevant for the fungible disutility case, mentioned in Section 6.1 because, there, $\delta_1' = \delta_2'$, regardless of the chosen ad volumes. In practice, heterogeneity in $r_i(\cdot)$ functions may arise if one website can provide advertisers better data about visiting users. For example, advertising exchanges have differential access to user data, and an ad impression is sold at a higher price when advertisers can learn its characteristics.[17] Thus if website 1 uses (or is run by) an advertising exchange that has preferential access to user data while website 2 does not, website 1 is likely to face a higher marginal revenue from selling ad impressions, i.e., $r_1'(\cdot) > r_2'(\cdot)$.

We conclude this section with two remarks. First, in Model 2, the analog to Assumption 1 is an assumption that $r_i(\alpha) = \rho_i \alpha^\ell$ and $\delta_i(\alpha) = \gamma_i \alpha^k$ with $k \geq 1 \geq \ell$ and $k \neq \ell$.[18] Then, the corresponding Model 1 features $\delta_i(r_i^{-1}(a)) = \gamma_i \rho_i^{-\frac{k}{\ell}} a^{\frac{k}{\ell}}$, which satisfies Assumption 1. Thus, under this analog assumption, all the results in Sections 3 and 4 extend. Second, we

[17] An advertising exchange operated by Google (e.g., Goole AdX) arguably has better access to user data (e.g., Srinivasan (2020)). Johnson et al. (2020) document that the price of an impression is low for a user who has opted out from behavioral targeting, indicating that the availability of data increases advertising revenue.

[18] If we assume $\delta_i(a) = \gamma_i a_i^k$ but keep r_i a general concave function in Model 2, then the corresponding Model 1 with disutility $\delta_i(r_i^{-1}(\cdot))$ may not satisfy Assumption 1.

can incorporate micropayments into Model 2. Website i now earns $r_i(\alpha_i) + t_i$ per visit, and consumers visit websites if and only if $v \geq C(\delta_1(\alpha_1) + \delta_2(\alpha_2)) + t_1 + t_2$. The equivalence between Models 1 and 2 may no longer hold, but, as we prove in Appendix E, it still is true that misplacement disappears when both websites can use micropayments.

7 Conclusion

Internet users frequently visit multiple ad-funded websites that play a complementary role in fulfilling their needs. This paper shows that, in such settings, websites have a tendency to distort their advertising policies in two ways. The first distortion is classic double marginalization, reflected by an excessive total amount of advertising. The second distortion is what we call *misplacement*: the websites could have jointly reallocated advertisement among themselves and increased their total profit, without changing consumers' payoffs.

In a standard vertical chain of monopolies, introducing intense competition at one level of the chain, either upstream or downstream, restores the industry optimum and improves efficiency. However, in our model, although intense competition among one type of complementary website eliminates double marginalization, it can also worsen the harms from misplacement. On the other hand, introducing micropayments, charged per-visit by websites to consumers, can correct misplacement, but not double marginalization. Moreover, the effects of adopting micropayments on consumer surplus and industry profit are ambiguous.

Understanding the costs and benefits of (de)-centralization in online advertising is important for platforms and society more broadly. This is particularly so for policymakers, including antitrust and legislative authorities, whose actions may benefit from solid economic reasoning applied to the attention economy, which we hope this paper helps provide.

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Appendix

A Existence of Equilibrium

We show the existence of an equilibrium in the baseline model. The game between the websites is an ordinal potential game with potential $a_1a_2D(\delta_1(a_1)+\delta_2(a_2))$ (Monderer and Shapley, 1996). That is, for any a_j, website i prefers a_i to a_i' if and only if $a_ia_jD(\delta_i(a_i) + \delta_j(a_j)) \geq a_i'a_jD(\delta_i(a_i') + \delta_j(a_j))$. A pure-strategy equilibrium exists if the potential function has a maximizer. Let $\overline{a}_i$ satisfy $\delta_i(\overline{a}_i) > \overline{v}$. The potential function, which is continuous, has a maximizer if the strategy space of each website i is restricted to the compact set $[0, \overline{a}_i]$. Even when the strategy space is not restricted, the maximizer continues to be an equilibrium of the original game because website i does not benefit from choosing $\overline{a}_i$, which leads to a payoff of zero. The potential function has a maximizer, so the game has an equilibrium. For the game with micropayments, we can apply the same argument to the potential $(a_1 + t_1)(a_2 + t_2)D(\delta_1(a_1) + \delta_2(a_2) + t_1 + t_2)$.

B Imperfect Complements

Here we relax the assumption that two websites are perfectly complementary. Consider the following version of the baseline model. There is a unit mass of consumers. A fraction $p \in [0, 1]$ of consumers, which we call group C, regard the two websites as perfect complements, as in the baseline model. Fraction $1 - p$ of consumers, which we call group N, do not face complementarity: They obtain a payoff of $v/2 - \delta_i(a_i)$ if they visit only website i, $v - \delta_1(a_1) - \delta_2(a_2)$ if they visit both websites, and zero if they visit none. The parameter p captures the degree of complementarity, and $p = 1$ corresponds to our baseline model. Users' values v are uniformly distributed between 0 and $\overline{v}$ and are independent of whether they face complementarity. We maintain Assumption 1, i.e., $\delta_i(a) = \gamma_i a^k$.

Double Marginalization

Claim B.1. *Equilibrium entails double marginalization if and only if $p > 0$. However, as p increases, total disutility increases and moves away from the industry optimum level, which is independent of p.*

Proof. First, we study equilibrium. The payoff to website i is

$$a_i \left\{ p[1 - F(\delta_i(a_i) + \delta_j(a_j))] + (1 - p)[1 - F(2\delta_i(a_i))] \right\}. \quad \text{(A.1)}$$

Note that the demand from group N is $1 - F(2\delta_i(a_i))$, because they visit website 2 if and only if $\frac{v}{2} \geq \delta_i(a_i)$. The first-order condition with respect to a_i is

$$\begin{aligned} & p[1 - F(\delta_i(a_i) + \delta_j(a_j))] + (1 - p)[1 - F(2\delta_i(a_i))] \\ & \qquad - pf(\delta_i(a_i) + \delta_j(a_j)) \cdot a_i \delta_i'(a_i) - (1 - p) f(2\delta_i(a_i)) \cdot 2a_i \delta_i'(a_i) = 0. \quad \text{(A.2)} \end{aligned}$$

Because $F = U[0, \overline{v}]$, we have $f = \frac{1}{\overline{v}}$. Assumption 1 implies $a_i \delta_i'(a_i) = k\delta_i(a_i)$. Thus we can write (A.2) as

$$p[\overline{v} - \delta_i(a_i) - \delta_j(a_j)] + (1-p)[\overline{v} - 2\delta_i(a_i)] - pk\delta_i(a_i) - (1-p) \cdot 2k\delta_i(a_i) = 0. \quad \text{(A.3)}$$

Summing the equations for $i = 1, 2$ and denoting $\delta_i(a_i) + \delta_j(a_j)$ as Δ^*, we obtain

$$\begin{aligned} & 2p(\overline{v} - \Delta^*) + (1-p)[2\overline{v} - 2\Delta^*] - pk\Delta^* - (1-p) \cdot 2k\Delta^* = 0 \\ \Rightarrow\ & \Delta^*(p) = \frac{2\overline{v}}{2 + k(2-p)}. \end{aligned} \quad \text{(A.4)}$$

Next, we show that the total disutility at the industry optimum is $\frac{\overline{v}}{1+k}$. Consider the maximization of the industry profit for $p = 0$ and $p = 1$. If $p = 0$, the industry optimum is $\Delta^*(0) = \frac{\overline{v}}{1+k}$, because there is no strategic interaction between the websites. If $p = 1$, the industry profit is

$$(a_1 + a_2)[1 - F(\delta_1(a_1) + \delta_2(a_2))].$$

The first-order condition is

$$a_1 + a_2 = \frac{g(\delta_1(a_1) + \delta_2(a_2))}{\delta_i'(a_i)}.$$

Because $\delta_1'(a_1) = \delta_2'(a_2)$, we have

$$\begin{aligned} & \delta_1'(a_1)a_1 + \delta_2'(a_2)a_2 = g(\delta_1(a_1) + \delta_2(a_2)) \\ \Rightarrow\ & \delta_1(a_1) + \delta_2(a_2) = \frac{1}{k} g(\delta_1(a_1) + \delta_2(a_2)) = \frac{1}{k}(\overline{v} - \delta_1(a_1) - \delta_2(a_2)) \\ \Rightarrow\ & \delta_1(a_1) + \delta_2(a_2) = \frac{\overline{v}}{1+k}. \end{aligned}$$

To sum up, both for $p = 0$ and $p = 1$, total disutility at the industry optimum is $\frac{\overline{v}}{1+k}$. Thus the same property holds for any $p \in [0, 1]$.

At $p = 0$, the equilibrium outcome maximizes the industry profit. As p increases, as is shown in (A.4), equilibrium total disutility increases and moves away from the industry optimum level. Thus, the distortion due to double marginalization is increasing in the degree of complementarity. $\square$

Although so far we have considered just two websites, the analysis extends to $N \geq 2$ websites. In this case, a share p of consumers each receives her v only by visiting all websites, while the remaining $1-p$ share each receives $v/N - \delta_i(a_i)$ for each website i she visits. We can write (A.3) as

$$p\left[\overline{v} - \sum_{i=1}^{N} \delta_i(a_i)\right] + (1-p)\left[\overline{v} - N\delta_i(a_i)\right] - pk\delta_i(a_i) - (1-p)\cdot Nk\delta_i(a_i) = 0.$$

Summing this equation for $i = 1, \ldots, N$ and denoting $\sum_{i=1}^{N} \delta_i(a_i)$ as Δ^*, we obtain

$$\begin{aligned} & Np(\overline{v} - \Delta^*) + (1-p)N[\overline{v} - \Delta^*] - pk\Delta^* - (1-p)\cdot Nk\Delta^* = 0 \\ \iff & N\overline{v} = \Delta^*(Np + N(1-p) + pk + (1-p)Nk) = \Delta^*(N + pk + (1-p)Nk) \\ \Rightarrow & \Delta^*(p) = \frac{N\overline{v}}{N + Nk - (N-1)kp} = \frac{\overline{v}}{1 + k - (1 - \frac{1}{N})kp}. \end{aligned}$$

As before, total disutility at the industry optimum continues to be $\frac{\overline{v}}{1+k}$. For a fixed p, as the number of websites, N, increases, total equilibrium disutility, $\Delta^*(p)$, moves further away from the industry optimum.

Misplacement

Claim B.2. *Take any $p > 0$. Equilibrium entails misplacement if and only if $\gamma_1 \neq \gamma_2$.*

Proof. Given the uniform distribution assumption on v, we can write industry profit as

$$(a_i + a_j)\cdot p\left[1 - \frac{\delta_i(a_i) + \delta_j(a_j)}{\overline{v}}\right] + (1-p)a_i\left[1 - \frac{2\delta_i(a_i)}{\overline{v}}\right] + (1-p)a_j\left[1 - \frac{2\delta_j(a_j)}{\overline{v}}\right]. \quad \text{(A.5)}$$

Fix $\Delta = \delta_i(a_i) + \delta_j(a_j)$ and let $\alpha \cdot \Delta = \delta_1(a_1)$ for some $\alpha \in [0, 1]$. Then we have $a_1 = \left(\frac{\alpha\Delta}{\gamma_1}\right)^{\frac{1}{k}}$

and $a_2 = \left(\frac{(1-\alpha)\Delta}{\gamma_2}\right)^{\frac{1}{k}}$. Thus, industry profit is

$$\Pi(\gamma_1, \gamma_2, \alpha, \Delta)$$

$$:= \left(\frac{\alpha\Delta}{\gamma_1}\right)^{\frac{1}{k}} \cdot p \left[1 - \frac{\Delta}{\overline{v}}\right] + (1-p)\left(\frac{\alpha\Delta}{\gamma_1}\right)^{\frac{1}{k}} \left[1 - \frac{2\alpha\Delta}{\overline{v}}\right] \tag{A.6}$$

$$+ \left(\frac{(1-\alpha)\Delta}{\gamma_2}\right)^{\frac{1}{k}} \cdot p \left[1 - \frac{\Delta}{\overline{v}}\right] + (1-p)\left(\frac{(1-\alpha)\Delta}{\gamma_2}\right)^{\frac{1}{k}} \left[1 - \frac{2(1-\alpha)\Delta}{\overline{v}}\right] \tag{A.7}$$

This expression for profits is strictly concave in α. In equilibrium, total disutility is Δ^*, and each website imposes the same amount of disutility, $\delta_i(a_i) = \frac{1}{2}\Delta^*$ If $\Delta = \Delta^*$, $\alpha = 1/2$, and $\gamma_1 = \gamma_2$, then the derivative of (A.6) with respect to α is positive, and that of (A.7) is negative.[19] Moreover, the absolute values of these first-order impacts are the same because the two websites are symmetric. Now if $\gamma_1 < \gamma_2$, the positive effect dominates the negative one, so $\alpha = 1/2$ cannot satisfy the first-order condition of industry profit maximization. Therefore, whenever $\gamma_1 = \gamma_2$, the equilibrium outcome $\alpha = 1/2$ cannot be industry-optimal. This implies that the equilibrium entails misplacement if and only if $\gamma_1 \neq \gamma_2$, for any $p > 0$. □

Competition

Even if we suppose that the distribution of v is uniform and adopt Assumption 1, we no longer have a result that perfect competition for one segment restores the industry-optimal level of total disutility. For example, if $p = 0$, then competition simply reduces industry profit. At the same time, we can extend the previous welfare comparison result.

Claim B.3. *Fix any $p \in [0, 1]$. Competition for website 2 increases the industry profit if and only if γ_2 exceeds some threshold, which can be ∞.*

Proof. The result holds because the industry profit after the introduction of competition is independent of γ_2, whereas, in the absence of competition, it decreases in γ_2. The latter observation

[19] For example, the derivative of (A.6) with respect to α is equivalent to the first-order condition of website 1 if we replaced the second Δ with $\alpha\Delta$; because the second Δ in (A.6) does not reflect the increase of α, the derivative of (A.6) must be positive when evaluated at the equilibrium objects.

holds because if γ_2 increases, total disutility remains the same but website 2 optimally chooses a lower a_2. $\square$

Micropayments

Similarly, micropayments may no longer eliminate misplacement when websites are imperfect complements, but we can extend the welfare implications to this case.

Claim B.4. *In the game with micropayments, equilibrium entails misplacement if and only if* $p \in (0,1)$.

Proof. As before, website i always sets a_i^* such that $\delta_i'(a_i^*) = 1$. Thus, the revenue per visit, given total disutility $\Delta_i = \delta_i + t_i$ from website i, is $\Delta_i + a_i^* - \delta_i(a_i^*)$. The equilibrium first-order condition is then

$$\Delta_i + a_i^* - \delta_i(a_i^*) = \frac{p[1 - F(\Delta_i + \Delta_j)] + (1-p)[1 - F(2\Delta_i)]}{pf(\Delta_i + \Delta_j)] + (1-p)f(2\Delta_i)}$$

or equivalently,

$$\Delta_i + a_i^* - \delta_i(a_i^*) = p[\overline{v} - \Delta_i - \Delta_j] + (1-p)[\overline{v} - 2\Delta_i], \tag{A.8}$$

which is written as

$$(3 - 2p)\Delta_i + a_i^* - \delta_i(a_i^*) = p[\overline{v} - \Delta_i - \Delta_j] + (1-p)\overline{v}. \tag{A.9}$$

If $\gamma_i < \gamma_j$, then $a_i^* - \delta_i(a_i^*) > a_j^* - \delta_i(a_j^*)$, so we have $\Delta_i < \Delta_j$ and $\Delta_i + a_i^* - \delta_i(a_i^*) > \Delta_j + a_j^* - \delta_i(a_j^*)$ in equilibrium. With v uniformly distributed, these inequalities imply that

$$\frac{\partial}{\partial \Delta_i}[\Delta_i + a_i^* - \delta_i(a_i^*)] \cdot [1 - F(2\Delta_i)] > \frac{\partial}{\partial \Delta_j}[\Delta_j + a_j^* - \delta_i(a_j^*)] \cdot [1 - F(2\Delta_j)].$$

The industry profit is

$$\begin{aligned}&(\Delta_i + a_i^* - \delta_i(a_i^*) + \Delta_j + a_j^* - \delta_j(a_j^*)) \cdot p\left[1 - \frac{\Delta_i + \Delta_j}{\overline{v}}\right] \\ &+ (1-p)(\Delta_i + a_i^* - \delta_i(a_i^*))\left[1 - \frac{2\Delta_i}{\overline{v}}\right] + (1-p)(\Delta_j + a_j^* - \delta_j(a_j^*))\left[1 - \frac{2\Delta_j}{\overline{v}}\right]\end{aligned}$$

The above inequality implies that industry profit increases if website 1 increases Δ_1 by $\varepsilon > 0$ and website 2 decreases Δ_2 by ε for a small $\varepsilon > 0$. □

Claim B.5. *Consumer surplus is greater in the game with micropayments than in the baseline model if and only if* $\sum_i \left(\frac{1}{\gamma_i}\right)^{\frac{1}{k-1}}$ *is above some threshold. For a sufficiently large* γ_1 *and* γ_2*, the introduction of micropayments increases the industry profit.*

Proof. If we sum up (A.8) for $i = 1, 2$, we obtain

$$\sum_i \Delta_i + \sum_i [a_i^* - \delta_i(a_i^*)] = 2p[\overline{v} - \sum_i \Delta_i] + (1-p)[2\overline{v} - 2\sum_i \Delta_i], \tag{A.10}$$

which implies

$$\sum_i \Delta_i = \frac{2\overline{v} - \sum_i [a_i^* - \delta_i(a_i^*)]}{3} = \frac{2\overline{v} - \left(1 - \frac{1}{k}\right)\left(\frac{1}{k}\right)^{\frac{1}{k-1}} \sum_i \left(\frac{1}{\gamma_i}\right)^{\frac{1}{k-1}}}{3}. \tag{A.11}$$

Thus, the equilibrium total disutility under micropayments is decreasing in $\sum_i \left(\frac{1}{\gamma_i}\right)^{\frac{1}{k-1}}$. Because total disutility without micropayments is independent of (γ_1, γ_2), we obtain the result for consumer surplus. The claim regarding the industry profit holds because, for a large γ_1 and γ_2, the industry profit without micropayments goes to zero, whereas the one with micropayments is bounded from below by $\frac{2\overline{v}}{3}\left[1 - F\left(\frac{2\overline{v}}{3}\right)\right]$. Here, $\frac{2\overline{v}}{3}$ is an upper bound for the equilibrium total disutility (from (A.11)) and a lower bounded for the per visit profit across all (γ_1, γ_2). □

C Appendix for Section 6: Microfoundation of Advertising Revenue

We microfound the advertising revenue of each website, following Anderson and Coate (2005). Then, we argue how the microfoundation fits the general model in Section 6.

Suppose there is a mass $m_1 + m_2$ of (potential) advertisers. Each advertiser is a monopolist of its product, which costs 0 to produce. For each $i \in \{1, 2\}$, the mass m_i of advertisers chooses between placing an ad on website i and taking an outside option that secures a payoff of zero. Below, pool i refers to the set of advertisers that consider placing an ad on website i. If an advertiser in pool i decides to place an ad on website i, it pays a price of p_i.

Consumers interact with ads in the following way. Suppose that a consumer visits website i. For each ad on website i, the consumer faces one of the following two events: (i) the consumer does not notice the ad or notices it but has no demand for the product, or (ii) the consumer notices the ad and has a positive value for the product. Which event occurs is independent across all consumers and all advertisers. Letting $\sigma_i \in [0, 1]$ denote the probability of event (ii), we assume that σ_i is drawn according to the cumulative distribution function G_i in each pool i. In event i, the consumer has a known (deterministic) value of $\theta_i > 0$ for the product. Thus, any advertiser sets a product price of θ_i upfront and extracts full surplus $\sigma_i\theta_i$.

An advertiser in pool i chooses to advertise if and only if $\theta_i\sigma_i \geq p_i$. The mass of potential advertisers for website i is m_i, and σ_i is drawn according to G_i. Thus, the mass of advertisers who advertise on website i is $a_i(p_i) := m_i(1 - G_i(p_i/\theta_i))$. The inverse demand is $p_i(a_i) := \theta_i G_i^{-1}(1 - a_i/m_i)$. On $[0, m_i]$, $p_i(\cdot)$ is positive and decreasing.

Each website is a quantity-setting monopolist in the advertising market, so given a_i, the price of an advertising slot is $p_i(a_i)$. We assume that each website i's advertising revenue per user, $r_i(a_i) := a_i p_i(a_i)$, is strictly concave on $[0, m_i]$ (e.g., G is a uniform distribution). The concavity implies that $r_i(\cdot)$ is uniquely maximized at some finite $\hat{a}_i < m_i$. Note that in Section 6, we have assumed $r_i(\cdot)$ is increasing. However, we can apply the same analysis by restricting each website's strategy space to $[0, \hat{a}_i]$. Indeed, any $a_i > \hat{a}_i$ is weakly dominated by $\hat{a}_i$, because $\hat{a}_i$ attains a higher demand and a higher per visit revenue than a_i.

D Appendix for Section 5: The Impact of Micropayments

Micropayments Can Decrease Industry Profits

Throughout the section, we impose Assumption 1. The following example shows that there is some $(k, \gamma_1, \gamma_2, F)$ such that the industry profit and consumer surplus decrease when the websites can use micropayments.

Example 1. Suppose $k = 3$, $\gamma_1 = \gamma_2 = 0.01$, and v is uniformly distributed between 0 and $\overline{v} = 10$. With micropayments, equilibrium total disutility Δ^P satisfies equation (13), which becomes

$$\Delta^P + \frac{40}{3} \cdot \frac{1}{\sqrt{3}} = 2(10 - \Delta^P),$$

which implies $\Delta^P = \frac{20(1-2\cdot\frac{1}{3\sqrt{3}})}{3} \approx 4.1$. The industry profit is then $2(10 - \Delta^P)(1 - \frac{\Delta^P}{10}) = 2 \cdot \frac{10+40\cdot\frac{1}{3\sqrt{3}}}{3} \cdot \frac{1+4\cdot\frac{1}{3\sqrt{3}}}{3}$. Without micropayments, the total disutility Δ^N solves $\Delta^N = \frac{2}{k} g(\Delta^N)$, which now becomes $\Delta^N = \frac{2}{3}(10 - \Delta^N)$. Thus we obtain $\Delta^N = 4 < \Delta^P$. Because each website chooses a such that $\delta_i(a) = 2$, we have $a = 200^{1/3}$. Thus the industry profit is $2 \cdot 200^{1/3} \cdot (1 - \frac{4}{10})$. Given the values for the industry profits and the total disutility with and without micropayments, we can numerically verify that the introduction of micropayments decreases the industry profit and consumer surplus.

Adoption of Micropayments by One Website

The following result regards the impact of one website, alone, adopting micropayments on the other website and on consumers. This result does not require v to be uniformly distributed.

Proposition 5. *Suppose Assumption 1 holds. Compare the situation in which (i) only website* 1 *can use micropayments, and (ii) no website can use micropayments. Consumer surplus under (i) is greater than that under (ii) if and only if* γ_1 *is below some threshold. Also, website* 2 *obtains a higher payoff under (i) than (ii) if and only if* γ_1 *is below some (possibly different) threshold. Moreover, if* γ_1 *is sufficiently small, the industry profit is greater under (i) than (ii).*

Proof. Suppose only website 1 uses micropayments, and let Δ denote the equilibrium total disutility. In equilibrium, a_2^* satisfies $a_2^* = \frac{g(\Delta)}{\delta_2'(a_2^*)}$, which reduces to

$$\delta_2(a_2^*) = \frac{g(\Delta)}{k}. \tag{A.12}$$

The first-order condition of website 1 is

$$\hat{\delta}_1 + \left(1 - \frac{1}{k}\right)\left(\frac{1}{k\gamma_1}\right)^{\frac{1}{k-1}} = g(\Delta).$$

Summing up the two equations, we obtain

$$\Delta + \left(1 - \frac{1}{k}\right)\left(\frac{1}{k\gamma_1}\right)^{\frac{1}{k-1}} = \left(1 + \frac{1}{k}\right) g\left(\Delta\right). \tag{A.13}$$

When no website uses micropayments, total disutility does not depend on (γ_1, γ_2) because of Assumption 1. After website 1 adopts micropayments, total disutility is increasing in γ_1. Also, equation (A.12) implies that website 2 chooses a higher a_2^* when Δ decreases. Thus a lower γ_1 increases consumer surplus and website 2's profit, which guarantees the existence of the thresholds. $\square$

The following example studies the setting where just one website (which we assume to be website 1) can charge micropayments. Below, we refer to this as the *partial micropayment* setting. This example shows that total industry profit can be lower under partial micropayments than in the baseline model without micropayments.

Example 2. Suppose $\gamma_1 = 0.01$, $\gamma_2 = 0.05$, $k = 2.9$, and v is uniformly distributed between 0 and $\overline{v} = 10$. Suppose only website 1 can charge micropayments, and let Δ_U denote equilibrium total disutility. Equation (A.13) implies that

$$\Delta_U = \frac{\overline{v}(1 + \frac{1}{k}) - (1 - \frac{1}{k})(\frac{1}{k\gamma_1})^{\frac{1}{k-1}}}{2 + \frac{1}{k}}$$

The payoff to website 1 is

$$\Pi_1 := g(\Delta_U)\left(1 - \frac{\Delta_U}{\overline{v}}\right) = (\overline{v} - \Delta_U)\left(1 - \frac{\Delta_U}{\overline{v}}\right).$$

The payoff to website 2 is

$$\Pi_2 := \left(\frac{g(\Delta_U)}{\gamma_2 k}\right)^{\frac{1}{k}}\left(1 - \frac{\Delta_U}{\overline{v}}\right) = \left(\frac{\overline{v} - \Delta_U}{\gamma_2 k}\right)^{\frac{1}{k}}\left(1 - \frac{\Delta_U}{\overline{v}}\right)$$

Under partial micropayments, equilibrium profit is $\Pi_1 + \Pi_2$, which we can numerically verify to be at least 5.8. In the total absence of micropayments, the analogous calculation as in Example 1 yields that the industry profit in equilibrium is at most 5.79. Thus, in this example, partial micropayments strictly reduce industry profits. We can also numerically verify that partial micropayments benefit both website 1 and consumers. Finally, in equilibrium under partial micropayments, we have $a_2^* \approx 3.6$ and $\delta_2'(a_2^*) \approx 1.7 > 1 = \delta_1'(a_1^*)$. Thus, misplacement persists in this setting.

We conclude this appendix by noting that a switch from the baseline model to a partial micropayments regime could exacerbate misplacement. To see this, suppose $\delta_1(\cdot) = \delta_2(\cdot) = \delta(\cdot)$. Without any micropayments, no equilibrium entails misplacement, because, given the symmetry of the game, the two websites choose the same ad volume. Under partial micropayments, as in (10), the first-order condition for website 1 is $\Delta_1^* + a_1^* - \delta(a_1^*) = g(\Delta_1^* + \delta(a_2^*))$. The first-order condition for website 2 is $a_2^* = \frac{g(\Delta_1^* + \delta(a_2^*))}{\delta'(a_2^*)}$. Suppose that the equilibrium does not entail misplacement, which implies $\delta'(a_2^*) = 1$ and thus $a_1^* = a_2^* = a^*$. Plugging these into website 2's first-order condition and combining this with website 1's, we obtain $\Delta_1^* = \delta(a_1^*)$, i.e., website 1 sets zero monetary transfer. Plugging back to website 1's first-order condition, we have $a^* = g(2\delta(a^*))$. Note that a^* is the solution of $\delta'(a^*) = 1$ and independent of F. Thus whenever distribution F fails to satisfy $a^* = g(2\delta(a^*))$, we obtain a contradiction, i.e., the equilibrium entails misplacement. For example, if F is the uniform distribution on $[0, \overline{v}]$, we have $g(x) = \overline{v} - x$. Except for the non-generic case of $\overline{v} = a^* + 2\delta(a_2^*)$, a switch to partial micropayments introduces misplacement.

E Appendix for Section 6

This appendix incorporates micropayments into the general model ("Model 2") of Section 6 and shows that no equilibrium entails misplacement. Consumers visit websites if and only if $v \geq C(\delta_1(\alpha_1)+\delta_2(\alpha_2))+t_1+t_2$. The payoff of website i is now $(r_i(\alpha_i)+t_i)\left[1 - F(C(\delta_i(\alpha_i) + \delta_j(\alpha_j)) + t_i + t_j)\right]$. The first-order conditions with respect to α_i and t_i are

$$
\begin{aligned}
&r_i'(\alpha_i)\left[1 - F(C(\delta_i(\alpha_i) + \delta_j(\alpha_j)) + t_i + t_j)\right] \\
-&(r_i(\alpha_i) + t_i) f(C(\delta_i(\alpha_i) + \delta_j(\alpha_j)) + t_i + t_j) \cdot C'(\delta_i(\alpha_i) + \delta_j(\alpha_j)) \cdot \delta_i'(\alpha_i) = 0, \\
&1 - F(C(\delta_i(\alpha_i) + \delta_j(\alpha_j)) + t_i + t_j) - (r_i(\alpha_i) + t_i) \cdot f(C(\delta_i(\alpha_i) + \delta_j(\alpha_j)) + t_i + t_j) = 0.
\end{aligned}
$$

Combining these equations, we obtain

$$
\frac{r_i'(\alpha_i)}{\delta_i'(\alpha_i)} = C(\delta_i(\alpha_i) + \delta_j(\alpha_j)). \tag{A.14}
$$

Because $y_i := \frac{r_i'}{\delta_i'}$ is decreasing, we can write this equation as $\alpha_i = y_i^{-1}[C(\delta_i(\alpha_i) + \delta_j(\alpha_j))]$, which implies $\delta_i(\alpha_i) = \delta_i\{y_i^{-1}[C(\delta_i(\alpha_i) + \delta_j(\alpha_j))]\}$. Summing this up for $i = 1, 2$ we obtain

$$
\delta_1(\alpha_1) + \delta_2(\alpha_2) = \delta_1\{y_1^{-1}[C(\delta_1(\alpha_1) + \delta_2(\alpha_2))]\} + \delta_2\{y_2^{-1}[C(\delta_1(\alpha_1) + \delta_2(\alpha_2))]\}, \tag{A.15}
$$

which uniquely determines $\delta_1(\alpha_1) + \delta_2(\alpha_2)$. To show there is no misplacement, let Δ^* denote the equilibrium level of total disutility. Consider the following problem:

$$
\begin{aligned}
&\max r_1(\alpha_1) + r_2(\alpha_2) + t_1 + t_2 \\
s.t. \quad &C(\delta_1(\alpha_1) + \delta_2(\alpha_2)) + t_1 + t_2 = \Delta^*.
\end{aligned}
$$

Plugging $t_1 + t_2 = \Delta^* - C(\delta_1(\alpha_1) + \delta_2(\alpha_2))$ into the objective and differentiating it with respect to α_1 and α_2, we obtain (A.14), which again determines $\delta_1(\alpha_1) + \delta_2(\alpha_2)$ as a solution of (A.15). As a result, if the websites jointly maximize revenue subject to the constraint that the total disutility $C(\delta_1(\alpha_1) + \delta_2(\alpha_2)) + t_1 + t_2$ is Δ^*, they end up choosing the equilibrium strategy. Therefore, websites cannot increase their revenue without changing the total disutility.