Addictive Platforms*

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Abstract

We study competition for consumer attention in which platforms can sacrifice service quality for attention. A platform can choose the "addictiveness" of its service. A more addictive platform yields consumers a lower utility of participation but a higher marginal utility of allocating attention. We provide conditions under which increased competition can harm consumers by encouraging platforms to offer low-quality services. In particular, if attention is scarce, increased competition reduces the quality of services because business stealing incentives induce platforms to increase addictiveness. Restricting consumers' platform usage may decrease addictiveness and improve consumer welfare. A platform's ability to charge for its service can also decrease addictiveness.

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1 Introduction

Online platforms, such as Facebook, Google, and Twitter, monetize consumer attention. Because attention is finite, competition for attention may encourage firms to improve their services to attract consumers. At the same time, there is a growing concern for consumers and policymakers—that competition for attention could also incentivize a firm to sacrifice its service quality for attention. For example, a platform may adopt news feeds that display low-quality content users are likely to watch; it may also adopt a certain user interface, such as an intrusive notification system or infinite scrolling (Scott Morton et al., 2019).[1]

We study a model of competition for consumer attention in which a platform can sacrifice service quality for attention. The model consists of a consumer and platforms. First, platforms choose the "addictiveness" of their services. Second, the consumer chooses the set of platforms to join, then allocates her attention. A more addictive platform yields the consumer a lower utility of participation but a higher marginal utility of allocating attention. As a result, the consumer prefers to join less addictive platforms, but after joining, she allocates more attention to more addictive platforms. The consumer incurs a cost of allocating attention. She also faces an attention constraint, which caps the maximum attention she can allocate. A platform provides the service for free and earns revenue that is increasing in the amount of attention the consumer allocates.

In our model, addictiveness captures a platform's choice to sacrifice the quality of its service to make it more capable of capturing consumer attention. For example, a social media company may adopt a certain user interface, such as the "thumbs up" on Facebook or the double ticks notification on WhatsApp, to promote social pressure and comparison, which may prolong app usage (Montag et al., 2019). An online game company may introduce loot boxes, which have attracted scrutiny because of a resemblance to gambling.[2] Such features may increase a platform's revenue from app

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[1]For example, Scott Morton and Dinielli (2020) argue that "another reduction in quality that Facebook's market power allows is the serving of addictive and exploitative content to consumers. Facebook deploys various methods to maintain user attention—so that it can serve more ads—using techniques that the medical literature has begun to demonstrate are potentially addictive."

[2]In video games, the loot box system is a feature that gives a player randomly selected virtual items. The psychology literature studies the relation among video games, loot boxes, and gambling addiction. For example, Brooks and Clark (2019) report that loot box engagement is correlated with gambling beliefs and problematic gambling behaviour in adult gamers. King and Delfabbro (2020) study video game addiction and state that "there is growing recognition that unrestricted screen time, particularly in younger people, can lead to harm and that gaming can be highly time-consuming and addictive for some vulnerable individuals."

usage and advertising by increasing the time consumers spend on services. However, consumers may not view them as an increase in quality.[3] As a first step to understand a potential interaction between market competition and the addictiveness of digital services, we adopt a rational framework and model an "addictive" service as a service that provides consumers with low utilities of participation and high marginal utilities of spending time.

Our main question is how increased competition affects platforms' behavior, service quality, and consumer welfare. Competition affects platforms' incentives in two ways. On the one hand, it encourages platforms to reduce addictiveness: If a consumer faces competing platforms, she loses less by refusing to join a single platform and continuing to use other services. To attract the consumer, a platform needs to reduce addictiveness and offer high service quality. On the other hand, competition introduces business stealing incentives, whereby a platform increases addictiveness to capture attention the consumer would allocate to its rivals.

The countervailing incentives derive our main insight: Under a certain condition, increased competition could encourage platforms to sacrifice quality for attention, leading to lower service quality and consumer welfare. Such an outcome is likely in particular when attention is scarce: When the attention constraint is tight, higher addictiveness does not increase total attention but only changes how the consumer divides her attention across platforms. Competition then introduces business stealing incentives, leading to higher addictiveness and lower consumer welfare. Conversely, if the consumer does not face a tight attention constraint, platforms that do not face competition set high addictiveness without discouraging consumer participation. Competition then incentivizes platforms to decrease addictiveness, leading to higher consumer welfare.

In the baseline model, platforms can adjust addictiveness at no cost. However, the main result holds even when each platform incurs the cost of increasing addictiveness, such as the cost of technological investment. In such a case, the equilibrium level of addictiveness can be too high even for platforms—i.e., they can increase profits by collectively reducing addictiveness. This result clarifies the gap between a platform's private and collective incentives to choose addictiveness.

As a policy remedy to improve service quality, we examine the impact of a digital curfew, which restricts the consumer's platform usage. For example, the Social Media Addiction Reduction Tech-

[3]For example, the introduction of loot boxes has received user backlash, which provides suggestive evidence that users do not necessarily view loot boxes as quality improvement. See, e.g., https://www.economist.com/business/2017/12/07/video-games-could-fall-foul-of-anti-gambling-laws.

nology Act (the "SMART" Act) proposed in the US requests that companies limit the time a user may spend on their services.[4] We model a digital curfew as a reduction of the consumer's attention capacity. A digital curfew may increase consumer welfare by limiting a platform's incentive to increase addictiveness.

Finally we examine the role of a platform's revenue model. We compare the baseline model to a model of price competition, in which platforms earn revenue only by charging prices that are independent of the level of attention. Because platforms do not monetize attention, they set zero addictiveness and offer high service quality. However, we also show that the consumer can be better off under attention competition: The consumer faces high marginal utilities from addictive services, so she can earn a high incremental gain by refusing to join a platform and continuing to use other services. Under a certain condition, the better outside option encourages platforms to offer higher net utilities to the consumer under attention competition than price competition.

Related literature The paper relates to the literature on platform competition, in particular competition for consumer attention (e.g., Rochet and Tirole 2003; Armstrong 2006; Anderson and De Palma 2012; Bordalo et al. 2016; Wu 2017; Evans 2017, 2019; Oehmke and Zawadowski 2019; Prat and Valletti 2019; Galperti and Trevino 2020; Anderson and Peitz 2020). Platforms in our model have a new strategic variable that captures a firm's choice to degrade quality for attention. We model such a choice as the increase in marginal utilities and the decrease in the level of utilities provided to consumers.[5] As a result, a platform in our model faces a trade-off between offering high-quality service to encourage participation and low-quality service to capture attention. The trade-off leads to the novel finding that increased competition benefits or harms consumers, depending on attention capacity, investment cost, and business models. The divergence between utilities and marginal utilities does not arise in competition on other dimensions, such as price or advertising load, in which they typically move in the same direction, or the allocation of attention

[4] See https://www.congress.gov/bill/116th-congress/senate-bill/2314 (accessed on January 6, 2022). Several other countries have implemented some restrictions to protect young people from addictive games. In 2003 Thailand implemented a shutdown law that banned young people from playing online games between 22:00 and 06:00. In 2011, South Korea passed a similar legislature, known as the Youth Protection Revision Act. In 2007, China introduced the so-called "fatigue" system under which game developers need to reduce or stop giving out rewards (e.g., game items, experience value) in games after a player reached certain hours of play.

[5] In this sense our model also relates to Armstrong and Vickers (2001), in which firms compete in utility space; in our model, platforms compete in the space of utilities and marginal utilities.

is not explicitly modeled (e.g., Anderson and Coate 2005; de Corniere and Taylor 2020; Choi and Jeon 2020).[6] The divergence also distinguishes our work from others that address competition in the presence of attention externalities, e.g., via the product complexity (Oehmke and Zawadowski, 2019) or most salient attributes (Bordalo et al., 2016).

Second, the paper contributes to the nascent literature on possible negative impacts of digital services on consumers (Allcott and Gentzkow, 2017; Allcott et al., 2020; Mosquera et al., 2020; Allcott et al., 2021). A recent discussion points out that technology companies may have an incentive to adopt features (e.g., user interfaces) that increase user engagement at the expense of their welfare (Alter, 2017; Scott Morton et al., 2019; Newport, 2019; Rosenquist et al., 2020). We contribute to this literature by examining interactions between competition for attention and the addictiveness of digital services. Although we later motivate our model based on habit formation with a time-inconsistent agent, we largely abstract away from dynamics and behavioral biases relevant to addiction (Becker and Murphy, 1988; Gruber and Köszegi, 2001; Orphanides and Zervos, 1995).

Finally, the recent policy and public debates recognize the problem that a firm that monetizes attention could distort its service quality to capture consumer attention (Crémer et al., 2019; U.K. Digital Competition Expert Panel, 2019; Scott Morton and Dinielli, 2020). We contribute to the discussion by providing a new intuition—that competition may not mitigate the problem.

2 Model

There are $K \in \mathbb{N}$ platforms and a single consumer. We write K for the number and the set of the platforms. Suppose that the consumer joins a set $J \subset K$ of platforms, and allocates attention $a_k \geq 0$ to each platform $k \in J$. If $J = \emptyset$, she receives a payoff of zero. Otherwise, her payoff is

$$\sum_{k \in J} u(a_k, d_k) - C\left(\sum_{k \in J} a_k\right). \tag{1}$$

[6] We abstract away from the two-sided aspect of the market, so our result differs from that of two-sided markets, in which competition for one side could harm other sides (e.g., Tan and Zhou 2021).

In the first term, $u(a_k, d_k)$ is the utility from platform k's service. The utility $u(a_k, d_k)$ depends on the *addictiveness* $d_k \in \mathbb{R}_+$ of platform k. We use subscripts to denote partial derivatives, such as $u_1 = \frac{\partial u}{\partial a}$ and $u_{12} = \frac{\partial^2 u}{\partial d \partial a}$. We impose the following assumption (see Figure 1).

Assumption 1. The function $u(\cdot,\cdot) : \mathbb{R}_+^2 \to \mathbb{R}$ is twice differentiable and satisfies the following:

(a) For every $d \geq 0$, utility $u(a, d)$ is strictly increasing and concave in a, and $u(0, 0) \geq 0$.

(b) For every $a \geq 0$, utility $u(a, d)$ is strictly decreasing in d, and
$\max_{a \geq 0}[u(a, d) - C(a)] < 0$ for some d.

(c) For every $a \geq 0$, the marginal utility for attention $u_1(a, d)$ is strictly increasing in d.

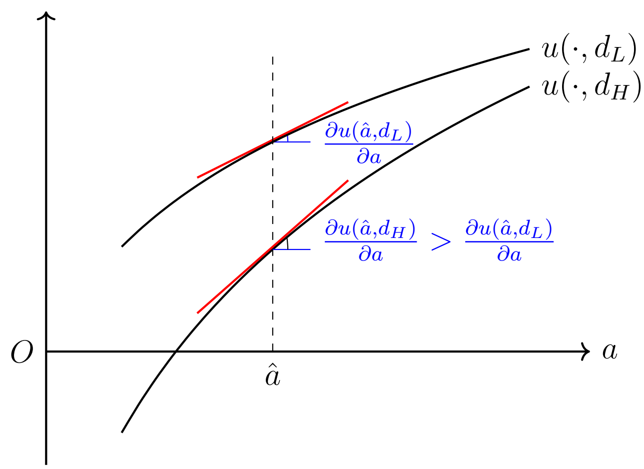


Figure 1: Utilities under d_L and $d_H > d_L$.

Points (b) and (c) imply that higher addictiveness decreases the consumer's utility of joining a platform but increases her marginal utility of allocating attention. Assumption 1 holds if, for example, $u(a, d) = 1 - e^{-\rho(a-d)}$ with $\rho > 0$ or $u(a, d) = v(a - d)$ with an increasing and concave $v(\cdot)$. Section 2.1 motivates the assumption.

The second term $C\left(\sum_{k \in J} a_k\right)$ of the consumer's payoff (1) is the *attention cost*—e.g., the opportunity costs of using digital services, such as benefits from consuming other services and goods or wages from working, as well as the cognitive cost of using platforms. We impose the following assumption.

Assumption 2. $C(\cdot) : \mathbb{R}_+ \to \mathbb{R}_+$ is strictly increasing, convex, and twice differentiable, and $\lim_{a \to 0}[u_1(a, 0) - C'(a)] > 0$.

The consumer also faces the *attention constraint*, which captures the scarcity of attention: She can allocate the total attention of at most $\overline{A} \in \mathbb{R}_+ \cup \{\infty\}$ across platforms. A finite $\overline{A}$ comes from, for example, a physical constraint or an exogenous restriction such as a digital curfew. Each of $\overline{A}$ and $C(\cdot)$ has its role: The attention constraint captures the scarcity of attention as a one-dimensional variable, and the attention cost creates a nontrivial choice of how much attention to allocate across digital services.[7] To ensure that an equilibrium exists, if $\overline{A} = \infty$, we assume that the primitives are such that the consumer has an optimal attention profile $(a_k)_{k \in J} \in \mathbb{R}^J_+$ given any $(d_1, \ldots, d_K)$ and any set $J \subset K$ of platforms she has joined (e.g., $u(\cdot, d)$ is bounded for each d). Remark 2 discusses the role of the attention constraint in detail.

If the consumer joins and allocates attention a to platform k, it earns a payoff of a. If the consumer does not join platform k, it receives a payoff of zero. A platform's payoff captures its advertising revenue, which increases in the time consumers spend on the platform.

The timing of the game is as follows: First, each platform $k \in K$ simultaneously chooses d_k, which the consumer observes. Second, the consumer chooses which platforms to join and how much attention to allocate. In equilibrium the consumer solves

$$\max_{J \subset K, (a_k)_{k \in J}} \sum_{k \in J} u(a_k, d_k) - C\left(\sum_{k \in J} a_k\right) \tag{2}$$

$$\text{subject to} \quad \sum_{k \in J} a_k \leq \overline{A} \quad \text{and} \quad a_k \geq 0, \forall k \in J.$$

The consumer obtains different utilities from platform k when she does not join k and when she joins it but chooses $a_k = 0$. The former gives her utility 0 whereas the latter gives her utility $u(0, d)$, which can be negative.[8] A negative $u(0, d)$ necessarily comes from the assumption that addictiveness d moves utilities and marginal utilities to the opposite directions. For example, if we instead set $u(0, d) = 0$ for all d, then we would have $u(a, d) = \int_0^a u_1(x, d)dx$, so utilities and marginal utilities would move to the same direction. In practice, a negative $u(0, d)$ reflects participation costs, such as efforts spent signing up for a platform or the cost of providing personal

[7]Our formulation is equivalent to a model in which the attention constraint does not exist, but the attention cost is such that $C(a) = \infty$ for all $a > \overline{A}$. We treat the attention cost and the attention constraint separately for ease of exposition.

[8]Assumption 1(b) and 1(c) imply that $u(0, d) < 0$ if and only if d exceeds some non-negative threshold.

data and losing privacy upon registration.

Our solution concept is pure-strategy subgame perfect equilibrium, which we call *equilibrium*. Under monopoly, we study an equilibrium in which the platform breaks ties in favor of the consumer. Remark 1 discusses the role of this tie-breaking assumption.

2.1 Interpretation of Addictiveness d

The addictiveness d captures the choice of a firm that makes its service more capable of capturing attention at the expense of quality. We capture such choices as the changes of service utilities and marginal utilities provided to consumers. The paper is agnostic about a particular mechanism that causes such changes. However, we present two applications that illustrate how the changes could occur.

2.1.1 Habit Formation

We motivate our utility specification using a three-period model of rational addiction with a time-inconsistent consumer (e.g., Becker and Murphy 1988; Gruber and Köszegi 2001). Given addictiveness $(d_1, \ldots, d_K)$, consider the following problem (see Figure 2). In $t = 1$, the consumer chooses the set $J \subset K$ of platforms to join. In $t = 2$, the consumer allocates attention $a_0 > 0$ and obtains utility $u_0 \geq 0$ on each platform in J. This period is a "pre-addiction" stage—i.e., the consumer has yet to be addicted, and the service utilities and the optimal amount of attention do not depend on $(d_1, \ldots, d_K)$.[9] In $t = 3$, the consumer allocates her attention across platforms in J. This period is a "post-addiction" stage: If the consumer allocates attention a to platform k, she receives $\hat{u}(a - a_0 d_k)$, where $\hat{u}(\cdot)$ is an increasing concave function with $\hat{u}(0) \geq 0$. The payoff $\hat{u}(a - a_0 d_k)$ captures linear habit formation (e.g., Rozen, 2010). Here, $a_0 d_k$ is the reference point against which the consumer evaluates service consumption of platform k in $t = 3$. We can interpret $\frac{1}{d_k}$ as the "rate of disappearance of the physical and mental effects of past consumption"

[9] We do not need to specify the utility function the consumer faces in $t = 2$. However, to derive our functional form, we need to assume that a_0 does not depend on the number of platforms the consumer has joined. One way to endogenize such an outcome is to assume that the consumer's utility from each platform in $t = 2$ is $v(a)$ that is maximized at an interior optimum $a_0 \leq \overline{A}/K$. Alternatively, we can assume that the utility function in $t = 2$ is $u(a, 0)$, the attention cost is linear ($C(a) = ca$), and the attention constraint does not bind in the pre-addiction stage, i.e., $\arg\max_{a \geq 0} u(a, 0) - ca \leq \overline{A}/K$.

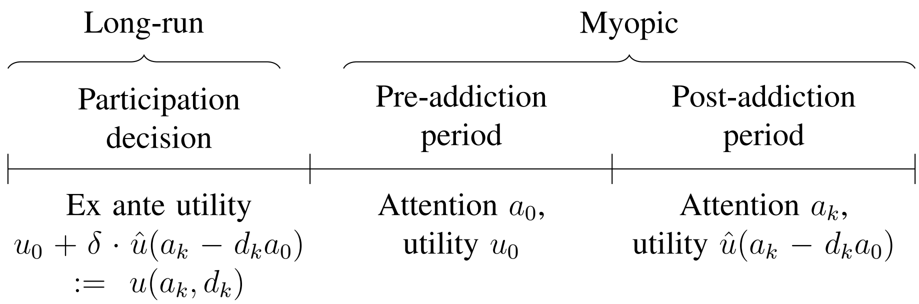


Figure 2: Three-period problem of the consumer

(Becker and Murphy, 1988). A higher d_k imposes a greater harm on the consumer in $t = 3$, and she needs to increase her attention in $t = 3$ to ensure the same payoff as in $t = 2$. Allcott et al. (2021) empirically show that consumption of digital services could exhibit habit formation.

Motivated by dual-self models, we assume that the long-run self makes the participation decision and the short-run selves allocate attention (e.g., Thaler and Shefrin, 1981; Fudenberg and Levine, 2006). Specifically, in $t = 1$ the long-run self decides which platforms to join, anticipating the behavior of future selves: In $t = 2$ the short-run self allocates attention a_0 to each platform, then in $t = 3$ she allocates attention $(a_k^*)_{k \in J}$ to maximize $\sum_{k \in J} \hat{u}(a_k - a_0 d_k) - C\left(\sum_{k \in J} a_k\right)$. Assume the long-run self has discount factor δ. The consumer's participation decision is based on the service utility $u(a_k, d_k) := u_0 + \delta \hat{u}(a_k - a_0 d_k)$, which satisfies Assumption 1.

Our model is suitable when a consumer is susceptible to addictive features of digital services, but she recognizes it and may avoid joining platforms as a commitment device. The model is not suitable for a consumer who joins platforms but can use them cautiously to avoid addiction. Such a situation would correspond to the consumer who is forward-looking in periods 2 and 3.

2.1.2 Data Collection and Personalization

Our model can apply to a situation that does not feature a typical "addiction." Suppose that a platform requests consumers to provide their personal data upon registration. Let d denote the amount of data the platform requests. To provide data, consumers incur a privacy cost of ℓd with $\ell > 0$. It captures negative consequences of data collection, such as the risk of data leakage, identity theft, and discrimination. The platform can use their data to personalize offerings, which increases the value of the service from the base value $w(a)$ to $(1+d)w(a)$, where $w(\cdot)$ is increasing, concave, and bounded. A consumer's utility from joining the platform is $u(a,d) := (1+d)w(a) - \ell d$. If $\ell > \sup_{a\geq 0} w(a)$, $u(a,d)$ satisfies Assumption 1. A consumer perceives a platform as low quality when it collects more personal information upon registration, but after joining it, she has more incentive to spend time on a platform that has more information about her.

2.2 Other Modeling Assumptions

Multi-homing. In our model, the consumer can join any set of platforms. In practice, consumers may divide time across social media, video streaming, mobile applications, and online games, all of which monetize attention. If $K \geq 2$ and the consumer could join at most one platform, she would join a platform with $d = 0$ in equilibrium.

Platform's revenue. We can generalize a platform's payoff function in two ways. First, the main insight holds even when a platform incurs a cost of raising d, which could be a cost of technological investment. Section 5 studies such a case. Second, most of the results continue to hold in the following setting: If the consumer allocates attention $(a_1, \ldots, a_K)$, platform k earns a payoff of $r_k(a_1, \ldots, a_K)$, where a function $r_k : \mathbb{R}_+^K \to \mathbb{R}$ is strictly increasing in a_k and depends arbitrarily on $(a_j)_{j\in K\setminus\{k\}}$. For example, a platform's payoff captures revenue in the advertising market, in which platforms can sell consumer attention at a market price.

Addictiveness reduces welfare. In practice, platforms may also adopt features that increase consumer attention and their welfare.[10] To incorporate such features, suppose the consumer's utility from a platform is $u(a,d,b)$, where $u(a,d,b)$ and $\frac{\partial u}{\partial a}(a,d,b)$ are increasing in $b \in [0,1]$. Since a higher b encourages the consumer to join a platform and allocates more attention, we can redefine $u(a,d) = u(a,d,1)$ and apply our model.

Representative consumer. A single consumer makes the model tractable and enables us to focus on our key forces with no potential confounding effects, such as consumer heterogeneity and network effects. In practice, consumers may be heterogeneous in their attention costs or tastes for platforms; consumers may also enjoy positive network effects on a platform. Although we exclude these forces, we expect that our main economic force would be relevant in broader settings with multiple

[10] Hagiu and Wright (2020) study a model of dynamic competition with data-enabled learning. In one specification, higher past consumption leads to greater consumption utilities in the future, which resembles beneficial addiction.

consumers.

3 Equilibrium

3.1 Monopoly ($K = 1$)

A monopolist maximizes attention subject to the consumer's participation constraint. Let $d^P(\overline{A})$ denote the highest addictiveness that satisfies the participation constraint, i.e., $d = d^P(\overline{A})$ uniquely solves $\max_{A\in[0,\overline{A}]} \; u(A,d) - C(A) = 0$. Let $A(d) := \arg\max_{A\geq 0} \; u(A,d) - C(A)$ denote the consumer's unconstrained choice of attention, which is independent of $\overline{A}$. We then define $d^A(\overline{A}) := \min\left\{d \in [0,\infty] : A(d) \geq \overline{A}\right\}$, which is the lowest addictiveness under which the consumer exhausts her attention. The following result characterizes the monopoly equilibrium and presents comparative statics with respect to attention cap $\overline{A}$.

Proposition 1. *In equilibrium the monopolist sets addictiveness* $\min\left\{d^A(\overline{A}), d^P(\overline{A})\right\}$*, which increases in* $\overline{A}$*. There is* $\overline{A}^M > A(0)$ *such that, as a function of* $\overline{A}$*, the consumer's equilibrium payoff is increasing on* $[0, A(0)]$*, decreasing on* $[A(0), \overline{A}^M]$*, and equal to zero on* $[\overline{A}^M, \infty]$*.*

Figure 3 depicts the equilibrium addictiveness and the consumer's equilibrium payoff under monopoly as a function of her attention capacity $\overline{A}$. A monopolist's incentive depends on the scarcity of attention. If the attention constraint is tight, the consumer exhausts her attention capacity $\overline{A}$ at zero addictiveness. In such a case, the monopolist sets $d = 0$. As $\overline{A}$ increases beyond $A(0)$, the monopolist increases addictiveness to incentivize the consumer to spend more attention. Although a higher $\overline{A}$ relaxes the attention constraint, the increased addictiveness reduces the service utility and harms the consumer. For a large $\overline{A} \geq \overline{A}^M$, the monopolist raises addictiveness to increase consumer attention until she becomes indifferent between joining and not joining the platform.

3.2 Competition ($K \geq 2$)

For each $K \geq 2$, define

$$A_K(d) := \arg \max_{A \in [0, \overline{A}]} Ku\left(\frac{A}{K}, d\right) - C(A).$$

The consumer chooses total attention $A_K(d)$ if she joins K platforms with addictiveness d. The following result characterizes the equilibrium.

Proposition 2. *Fix any $K \geq 2$. In a unique equilibrium, all platforms choose addictiveness d^* that makes the consumer indifferent between joining and not joining each platform:*

$$K \cdot u\left(\frac{A_K(d^*)}{K}, d^*\right) - C\left(A_K(d^*)\right) = (K-1) \cdot u\left(\frac{A_{K-1}(d^*)}{K-1}, d^*\right) - C\left(A_{K-1}(d^*)\right). \quad (3)$$

Moreover, the equilibrium addictiveness d^* is always positive.

The intuition for the first part is as follows. Upon choosing addictiveness, each platform faces a trade-off. On the one hand, higher addictiveness renders its service less attractive to the consumer. On the other hand, conditional on joining, she will allocate more attention to more addictive services. Each platform then prefers to increase its addictiveness so long as the consumer joins it. The equilibrium addictiveness thus makes the consumer indifferent between joining and not joining each platform.

The second part states that the equilibrium addictiveness is always positive. The reason is as follows. At $d = 0$, a platform yields the consumer a non-negative payoff. Also, platforms' services are differentiated, in that they offer concave utilities. As a result if all platforms set $d = 0$, the consumer strictly prefers to join all platforms. However, a platform can then profitably deviate by slightly raising d_k: So long as the increment of d_k is small, the consumer continues to join all platforms. However, the consumer now allocates more attention on platform k and less attention on other platforms, because she faces higher marginal utilities on platform k than other platforms. Therefore, the equilibrium addictiveness d^*, which renders such a deviation unprofitable, is positive. This argument holds even if greater addictiveness does not change the total attention. For example, suppose that $C(\cdot) \equiv 0$, so that the consumer's total attention is always $\overline{A}$. Each platform still prefers to set $d > 0$ to capture a greater fraction of $\overline{A}$. In such a case, platforms could collectively decrease d to increase consumer surplus without changing their profits.

4 The Impact of Competition

We now turn to the main question: How does increased competition affect platforms' behavior and consumer welfare? We consider the question by analyzing two notions of increased competition: comparison of monopoly and duopoly, and comparison between the equilibrium and the joint-profit maximizing outcome.

4.1 Monopoly vs. Duopoly

To begin with, we compare monopoly to duopoly. At the same level of addictiveness, the consumer prefers duopoly because she can use more services. When platforms choose addictiveness, the impact of competition depends on the scarcity of attention. Recall that $A(0)$ is the consumer's attention choice on a monopoly platform with zero addictiveness, and $\overline{A}^M$ is a threshold such that for any $\overline{A} \geq \overline{A}^M$, consumer surplus is zero under monopoly.

Proposition 3. *If attention is so scarce that $\overline{A} \leq A(0)$ holds, then duopoly platforms choose higher addictiveness than the monopolist, and the consumer is strictly better off under monopoly. If $\overline{A} \geq \overline{A}^M$, then duopoly platforms choose lower addictiveness than the monopolist, and the consumer is weakly better off under duopoly.*

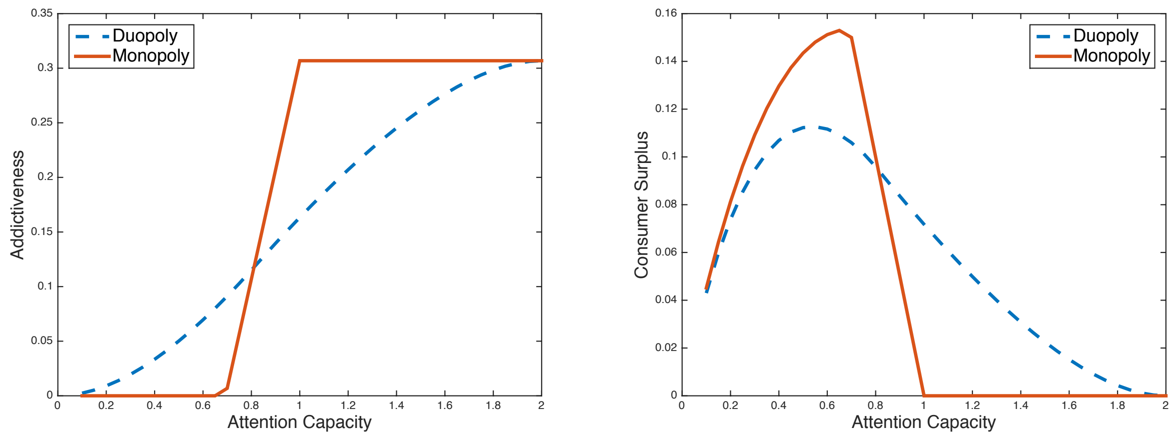
Proof. If $\overline{A} \leq A(0)$ the monopolist chooses zero addictiveness (Proposition 1). Under duopoly the consumer's payoff equals the payoff from joining a single platform, which now chooses positive addictiveness. As a result, the consumer is strictly better off under monopoly. If $\overline{A} \geq \overline{A}^M$, the consumer receives a payoff of zero under monopoly but a non-negative payoff under duopoly. $\square$

Figure 3 depicts the equilibrium addictiveness and the consumer's payoffs under monopoly and duopoly as a function of $\overline{A}$. When $\overline{A}$ is below $A(0)$, the consumer exhausts her attention $\overline{A}$ at zero addictiveness, so the monopolist sets $d = 0$. Under duopoly, however, a platform benefits from higher addictiveness because the consumer will allocate a greater fraction of her total attention to more addictive services. Thus competition for attention increases addictiveness and decreases consumer surplus. Suppose now that we increase $\overline{A}$ from $A(0)$. Both monopoly and duopoly platforms start to increase d in response to a higher $\overline{A}$, in order to encourage the consumer to spend more of her available attention. When $\overline{A}$ is sufficiently large, the monopolist degrades service quality so much that the consumer is indifferent between joining and not joining the platform. The same quality degradation does not necessarily occur under duopoly, because if a platform did so, the consumer might refuse to join the platform and instead spend her time on its competing service.[11] Thus for a large $\overline{A}$, both monopoly and duopoly platforms choose high addictiveness, but duopolists choose a (weakly) lower d than a monopolist.

Compared to monopoly, business stealing incentives encourage duopoly platforms to increase addictiveness, but competition for consumer participation discourages platforms from doing so. In the example of Figure 3, the latter beneficial impact of competition dominates business stealing incentives if and only if $\overline{A}$ exceeds a threshold. As a result, there is a unique $\overline{A}$ above which duopoly yields a greater consumer surplus than under monopoly. However, in general, how the magnitudes of these effects respond to an increase in $\overline{A}$ could be ambiguous. Correspondingly, Proposition 3 is silent about intermediate values of $\overline{A}$.

The above discussion provides an alternative interpretation of attention cap $\overline{A}$. We initially interpret $\overline{A}$ as a primitive that restricts the consumer's choice, such as a time constraint. However, the key driver of Proposition 3 is that when $\overline{A}$ is low, the consumers' total attention becomes irresponsive to an increase of addictiveness at a relatively low level of d. For example, if $\overline{A} \leq A(0)$, higher addictiveness does not change total attention. In such a case, a platform raises addictiveness not to increase total attention but to capture a larger faction of total attention, which makes it more likely that competition increases d and decreases service quality. Therefore, we can also interpret $\overline{A}$ as a parameter that reflects the effectiveness of addictive features in prolonging the consumer's total service usage, such as technological constraints.

[11] Figure 3 assumes a linear attention cost, in which case the consumer's payoffs under duopoly also becomes zero for a sufficiently large $\overline{A}$. If $C(\cdot)$ is linear and $\overline{A}$ is sufficiently large, joining one platform does not increase the cost for the consumer of joining other platforms. Thus the consumer's participation and attention allocation become separable across platforms. Platforms then act as monopolists, setting d that makes the consumer indifferent between joining and not joining.



(a) Equilibrium addictiveness

(b) Consumer surplus

Figure 3: Monopoly and duopoly equilibria under $u(a, d) = 1 - e^{-(a-d)}$ and $C(a) = 0.5a$ with varying attention capacity $\overline{A}$.

4.2 Equilibrium vs. Joint-Profit Maximizing Outcome

In this section, we study the impact of competition by comparing the equilibrium to the joint-profit maximizing outcome, in which K platforms bundle their services and act to maximize the total profits. The joint-profit maximizing outcome captures the lack of competition without changing the number of platforms in the market. Unlike the comparison of equilibria with different numbers of platforms, this approach enables us to study how increased competition affects market outcomes through the choice of addictiveness but not through the increased service variety.

Definition 1. The *joint-profit maximizing outcome* is the equilibrium of the game in which the platforms collectively choose $(d_1, \ldots, d_K)$ to maximize the sum of their profits while breaking ties for the consumer, and she only chooses between joining all platforms and joining none.

Proposition 4. *The following holds.*

1. *For any $K \geq 2$, there is an $A^* > 0$ such that if $\overline{A} \leq A^*$, each d_k is strictly lower and the consumer's payoff is strictly higher under the joint-profit maximizing outcome than the equilibrium.*

2. *Suppose $\lim_{a \to 0} u_1(a, 0) = \infty$. For any $\overline{A} \in \mathbb{R}_{++}$ there is a K^* such that if $K \geq K^*$, each d_k is strictly lower and the consumer's payoff is strictly higher under the joint-profit maximizing outcome than the equilibrium.*

3. *For any $K \geq 2$, there is an $A^{**} > 0$ such that if $\overline{A} \geq A^{**}$, each d_k is weakly lower and the consumer's payoff is weakly higher at the equilibrium than the joint-profit maximizing outcome.*

The intuition for Points 1 and 3 is similar to that of Proposition 3. Relative to the joint-profit maximizing outcome, competition encourages a platform to offer a higher service quality, because otherwise, the consumer can refuse to join a platform and use other services. At the same time, competition introduces business stealing incentives. If $\overline{A}$ is low, increasing addictiveness does not change the consumer's total attention, but only changes how she divides her attention across platforms. Thus for a small $\overline{A}$, the business stealing incentives become a dominant force, and competition lowers consumer welfare. Point 2 implies that even if we arbitrarily fix $\overline{A}$, the same welfare result holds for a large K under the Inada-type condition. A larger K works similarly as a smaller $\overline{A}$, because it tightens the attention constraint relative to the number of available services.[12]

We conclude this section by noting that the equilibrium behavior of competing platforms resembles the classic tragedy of the commons: Attention is a shared resource, and platforms exploit it by increasing d.[13] The business stealing incentives we highlight resemble the incentives that cause the "over-exploitation" of the common resource. However, a difference between our model and the tragedy of the commons is the consumer's endogenous participation: Unlike a shared resource, the consumer is a player and can avoid joining a platform that offers low utilities. Endogenous participation provides an intuition that is absent in the tragedy of the commons. For example, if a single user owns a resource, the over-exploitation of the resource does not occur. An analogous result may fail in our setting: A monopolist may choose a greater d than competing platforms, which choose a low d to encourage consumer participation. This tension between attention and participation is absent in the classic tragedy of the commons.

Remark 1 (Tie-breaking assumption). We have assumed that under monopoly (or at the joint-profit maximizing outcome), the platform breaks a tie in favor of the consumer. Our result—that

[12] Analogous results hold for other variables. For example, suppose that $\overline{A} < \infty$ and that the consumer's attention cost takes the form of $\epsilon C(\cdot)$. For a sufficiently small $\epsilon > 0$, the consumer's attention constraint binds at $d = 0$, in which case she is better off under the joint-profit maximizing outcome than the equilibrium.

[13] Our paper is not the first that relates firms' competition for attention to the tragedy of the commons. Oehmke and Zawadowski (2019) offers such an argument in a model in which firms choose complexity of their products to grab consumer attention. The difference between our model and the classic tragedy of the commons we discuss here also applies to the difference between our paper and their work.

competition can increase addictiveness—relies on this assumption. To see this, assume instead that the monopolist breaks ties to *minimize* the consumer's payoff by selecting the highest optimal d. In such a case, the monopolist sets d to make the consumer indifferent between joining and not joining the platform, and thus competition benefits the consumer even for a small $\overline{A}$. However, we can drop this tie-breaking assumption when platforms incur costs to increase addictiveness, in which case the equilibrium is unique under both monopoly and competition. The next section studies such a setting.

5 Costly Investment in Addictive Technology

This section extends the analysis by assuming that a platform incurs the cost of raising addictiveness, such as the cost of technological investment. Throughout this section, we impose the following assumptions.

Assumption 3. Each platform incurs a cost of $\kappa\gamma(d)$ to choose d with $\kappa > 0$. The function $\gamma : \mathbb{R}_+ \to \mathbb{R}_+$ is increasing, strictly convex, and differentiable, and satisfies $\lim_{d\to 0} \gamma'(d) = 0$ and $\lim_{d\to\infty} \gamma'(d) = \infty$. Assume $u(a, d) = v(a - d)$, and $g'(C'(x))C''(x)$ is weakly decreasing in x, where $g(\cdot)$ is the inverse of $v'(\cdot)$.

The last condition makes the payoff of platform k concave in d_k given the consumer's optimal behavior. The condition holds, for example, if $C(\cdot)$ is quadratic. The parameter κ captures how costly it is for a platform to increase addictiveness, and $\kappa = 0$ is our baseline model.

We have shown that for a large $\overline{A}$, competition benefits the consumer because a platform that has market power will choose a high d to increase attention. The costly choice of addictiveness may overturn this result.[14] The following result shows that if the cost of increasing d is substantial, increased competition, in the sense we studied in Section 4.2, leads to higher addictiveness regardless of $\overline{A}$.[15]

[14] The other result—i.e., increased competition reduces consumer welfare if $\overline{A}$ is small—continues to hold for any $\kappa \geq 0$.

[15] Proposition 5 does not contradict Proposition 4. Even in the current model, for a sufficiently small $\overline{A}$, the consumer is better off under the joint-profit maximizing outcome for any $\kappa \geq \kappa^* = 0$, which conforms Proposition 4.

Proposition 5. *Assume each platform incurs cost $\kappa\gamma(d)$ to choose addictiveness d. There is a unique equilibrium in which all platforms choose the same, positive addictiveness. For any $\overline{A} > 0$ including $\overline{A} = \infty$, there is some $\kappa^* \in \mathbb{R}_+$ such that each d_k is lower and consumer surplus is higher under the joint-profit maximizing outcome than the equilibrium if and only if $\kappa \geq \kappa^*$.*

To see the intuition, suppose for simplicity that $\overline{A} = \infty$. If κ is low, under the joint-profit maximization, platforms choose a high d that yields the consumer a payoff of zero. In such a case, competition benefits the consumer for the standard reason: The consumer can choose to not join a low-quality platform and to use other services. The better outside option incentivizes platforms to decrease addictiveness, leading to a higher consumer welfare. In contrast, for a high κ, a platform's choice is determined not by the consumer's participation incentive but by the marginal calculus between the benefit of more attention and the cost of increasing d. In such a case, competing platforms, which have business stealing incentives, sacrifice quality for attention more than how they would in the absence of competition.

Proposition 5 also implies that platforms benefit from collectively reducing addictiveness. To see this, suppose we have $\kappa \geq \kappa^*$, in which case the equilibrium addictiveness exceeds the joint-profit maximizing level, d^J. Suppose that platforms collectively cap the maximum level of addictiveness at d^J, which yields a game where the strategy space of each platform is restricted to $[0, d^J]$. In the equilibrium of this modified game, platforms set addictiveness d^J and the consumer joins all platforms.[16] Platforms attain the joint-profit maximizing outcome, and thus each platform obtains a higher payoff than without the cap. Therefore, platforms can increase their profits with a collective self-regulation that imposes a cap on the maximum level of addictiveness (see also the discussion in Section 8). We summarize this observation below:

Corollary 1. *Suppose $\kappa \geq \kappa^*$, and let d^J denote the level of addictiveness at the joint-profit maximizing outcome. The equilibrium payoff of each platform is greater when each platform can choose the addictiveness of at most d^J than when platforms can choose any levels of addictiveness.*

Remark 2 (Relaxing the attention constraint). We may think that some of our results so far rely on a "hard" attention constraint, whereby the consumer incurs infinite marginal costs at $\overline{A}$.

[16] The modified game is different from the problem of joint-profit maximization, because the consumer in the former can still choose which platforms to join. The consumer joins all platforms at d^J because she would do so when platforms set $d^* > d^J$, and her incremental gain of joining a platform is, by Lemma B.2, non-increasing in d.

A natural question is whether our analysis extends to the case in which the consumer incurs a sufficiently high but finite marginal attention cost around $\overline{A}$. Such an extension preserves the main insight if (and only if), as in this section, platforms incur costs to increase addictiveness.[17] To see this, suppose that the consumer does not face the attention constraint, but she has attention cost $C(a) = \hat{C}(a) + \mathbf{1}_{\{a \geq \overline{A}\}} \cdot \frac{c}{2}(a - \overline{A})^2$, where $\hat{C}$ satisfies Assumption 2. If c is large, the consumer faces greater costs and marginal costs of spending attention above $\overline{A}$. If $c = \infty$, attention cost C subsumes the attention constraint $A \leq \overline{A}$. Each platform incurs a strictly increasing, convex cost of raising d. Appendix E shows that if $\overline{A}$ is small enough to satisfy $u_1(\frac{\overline{A}}{K}, 0) - \hat{C}'(\overline{A}) \geq 0$, then for a sufficiently large c, consumer surplus is strictly greater under monopoly than under competition. Intuitively, when c is large, a higher d does not affect the consumer's total attention, but only affects the division of attention across platforms. Thus competing platforms, which have business stealing incentives, face greater marginal gain of raising d than the monopolist. Therefore, provided that platforms incur costs to raise d, we can replace the attention constraint with a finite attention cost that becomes large once it exceeds $\overline{A}$.

Remark 3 (Competition in beneficial addictive features). The crucial assumption for our results is that a platform has to sacrifice quality for attention, which we capture by assuming that utilities are increasing and marginal utilities are decreasing in d. To appreciate the role of this assumption, it is instructive to compare our model to a model in which both u and $\frac{\partial u}{\partial a}$ are increasing in d. Appendix F solves such a model, assuming that a platform incurs costs to increase d. In this model, higher d not only induces consumer participation but also increases the consumer's attention. Thus platforms no longer face the trade-off between quality and attention, and platforms choose higher d in equilibrium than the joint-profit maximizing outcome. As a result, regardless of C and $\overline{A}$, competition improves service quality.

[17]Some of our results fail if platforms can increase d at no cost, as in Section 4. If $\overline{A} = \infty$, a monopoly platform chooses d that makes the consumer indifferent between joining and not joining the platform, regardless of the size of (finite) marginal attention costs. In such a case, the consumer is always weakly better off under competition.

6 Digital Curfew

How would a regulator increase consumer welfare when it cannot directly control a platform's choice of d? We examine the impact of a digital curfew, which restricts the consumer's platform usage. Under a digital curfew at A, the consumer's attention cap becomes $\overline{A} = A$. Recall $A(0)$ denotes the consumer's optimal attention on a monopoly platform with zero addictiveness. For simplicity, we assume that platforms can choose any d at no cost.

Proposition 6. *The following holds.*

1. *In a monopoly market, a digital curfew at $\overline{A} = A(0)$ ensures the consumer-optimal outcome.*
2. *For any $K \geq 2$, the equilibrium addictiveness is positive at any level of digital curfew, and thus no digital curfew attains the consumer-optimal outcome.*
3. *Take any $K \geq 2$ and $\overline{A} = A$. Suppose the attention constraint holds with a strict inequality in equilibrium. Then a digital curfew at some $\overline{A} = A_D < A$ strictly decreases the equilibrium addictiveness and benefits the consumer.*

A digital curfew reduces a platform's incentive to increase addictiveness to expand the consumer's total attention, but it does not eliminate business stealing incentives. For example, consider a digital curfew at $\overline{A} = A(0)$, which prevents the consumer from spending longer time on digital services than how much she would have spent if the services had zero addictiveness. Under monopoly, such a digital curfew makes it optimal for the platform to set zero addictiveness. The same digital curfew, however, does not eliminate business stealing incentives, because the consumer will allocate a greater fraction of her attention to more addictiveness platforms, even if the total attention is fixed.

Two remarks are in order. First, we have examined a policy that limits total attention across platforms, but it is not the only way to define a digital curfew. For example, suppose a regulator could require that the consumer spend at most $A_K(0)/K$ unit of attention (defined in Section 3.2) on *each* platform. Such a policy would induce zero addictiveness and maximize consumer welfare.

Second, we assume that a digital curfew is exogenous to the consumer, but we could ask whether consumers are willing to adopt a digital curfew voluntarily. Suppose that there is a continuum of consumers, each of whom $i \in [0,1]$ chooses the maximum amount of attention $A_i \in [0, A_{max}]$ she can spend on platforms ($A_{max} > 0$ is an exogenous cap on possible attention constraints). After consumers choose $(A_i)_{i\in[0,1]}$, the original game of attention competition is played.[18] In equilibrium, all consumers choose the maximum attention A_{max}, because each consumer is atomless and her choice does not affect the behavior of platforms. Consumers cannot voluntarily enforce a digital curfew, even though they could benefit from collectively reducing A_i's.

7 The Role of Revenue Models

We have shown that competition can benefit or harm the consumer. The result depends on the revenue model of platforms. To highlight the idea, we study the following model of price competition and compare it to our original model. First, each platform $k \in K$ simultaneously chooses its addictiveness $d_k \geq 0$ and price $p_k \in \mathbb{R}$. The consumer observes $(d_k, p_k)_{k\in K}$, then chooses the set $J \subset K$ of platforms to join and how much attention to allocate. The consumer pays price p_k to join platform k. Each platform $k \in J$ receives a payoff of p_k, and any platform $k \notin J$ obtains a payoff of zero. The consumer receives a payoff of $\sum_{k\in J}[u(a_k, d_k) - p_k] - C\left(\sum_{k\in J} a_k\right)$ if $J \neq \emptyset$ and zero if $J = \emptyset$. In equilibrium, the consumer solves

$$\max_{J\subset K,(a_k)_{k\in J}} \sum_{k\in J}[u(a_k, d_k) - p_k] - C\left(\sum_{k\in J} a_k\right) \tag{4}$$

$$\text{subject to} \quad \sum_{k\in J} a_k \leq \overline{A} \quad \text{and} \quad a_k \geq 0, \forall k \in J,$$

where the objective is zero if $J = \emptyset$. Under price competition, platforms do not monetize attention, and they charge prices that are independent of consumer attention. The model captures digital services not supported by advertising, such as Netflix and YouTube Premium.[19]

Lemma 1. *The game of price competition has a unique equilibrium, in which all platforms choose*

[18] If platform k obtains attention a_k^i from each consumer i, then k's profit is $\int_{i\in[0,1]} a_k^i$.

[19] We do not consider the endogenous choice of business models or richer pricing instruments that may use allocated attention to determine a price. For recent studies on business models in two-sided markets, see, e.g., Gomes and Pavan (2016), Lin (2020), Carroni and Paolini (2020), and Jeon et al. (2021).

zero addictiveness and set the same positive price that makes the consumer indifferent between joining K and $K-1$ platforms. Consumer surplus is minimized under monopoly.

Under price competition the profits of platforms do not depend on attention, so they prefer to decrease addictiveness and charge higher prices. In equilibrium all platforms set zero addictiveness, and the equilibrium price p^* equals the incremental contribution of each platform to the consumer's total payoff. In particular, a monopoly platform extracts full surplus. As a result, under price competition, increased competition examined in Section 4 benefits the consumer.

We now compare different business models from the consumer's perspective. The consumer can use services for free under attention competition, but the service quality is typically lower than under price competition. The following result shows that the consumer can be better off under attention competition. Thus, higher service quality under price competition may not translate into higher consumer surplus.

Proposition 7. *Suppose that the supply of total attention is inelastic—i.e., $C(\cdot) \equiv 0$ and $\overline{A} < \infty$. If $K \geq 2$, the consumer is strictly better off in equilibrium under attention competition than price competition.*

The intuition is as follows. Consider duopoly, and take an equilibrium under attention competition or price competition. Let v^* denote the consumer's equilibrium payoff on each platform. Let v denote the payoff she could obtain by joining only one platform and allocating attention optimally. In equilibrium, the consumer joins both platforms and obtains payoff $2v^*$. Under either revenue model, the consumer is indifferent between joining one and two platforms, i.e., $v = 2v^*$. Thus the equilibrium payoff on each platform is written as $v^* = 2v^* - v^* = v - v^*$, and her total payoff is $2(v - v^*)$. Under price competition, $v - v^*$ is $u(\overline{A}, 0) - u(\frac{\overline{A}}{2}, 0)$, because platforms set $d = 0$ and the prices in v and v^* are canceled out. Under attention competition, $v - v^*$ is $u(\overline{A}, d^*) - u(\frac{\overline{A}}{2}, d^*)$. The consumer faces a higher marginal utility under a higher d, so we have $u(\overline{A}, d^*) - u(\frac{\overline{A}}{2}, d^*) > u(\overline{A}, 0) - u(\frac{\overline{A}}{2}, 0)$. Thus the consumer is better off under attention competition. The consumer's equilibrium payoff is proportional to the gain $v - v^*$ that she can earn if she stops using one platform and spends the saved time on the other platform. If this gain is high, platforms have to offer higher utilities by setting a lower d or p to attract the consumer. The gain is greater when a platform is more "addictive" in the sense we formulated.

8 Managerial Implications and Conclusion

We begin to wrap up this paper by discussing some managerial implications of our results. First, a model of competition between firms typically considers choice variables—such as quality and price—that simultaneously increase or decrease the utilities of joining a service and utilities of spending time on it. However, a recent discussion on "digital addiction" focuses on services that are apparently low quality but capable of capturing consumer attention. We model such a service as an "addictive" service and clarify a firm's trade-off between quality and attention. When a certain feature degrades the quality of service but increases the time users spend, a firm will face a trade-off between encouraging participation and capturing attention. We have highlighted the variables that could affect a platform's optimal level of addictiveness, such as the responsiveness of consumer attention and the cost of investment with respect to higher addictiveness.

Second, we have shown that the equilibrium addictiveness can be too high even from platforms' perspective (e.g., Corollary 1). The result indicates the potential benefit for platforms of identifying and collectively self-regulating addictive features that degrade quality. For example, platforms may adopt a certain user interface such as giving options to hide the number of "likes" or stop infinite scrolling. Alternatively, a platform may acquire another platform to internalize how the design of one service affects consumers' attention allocation on other services. We have also shown that business models affect the optimal level of addictiveness. In particular, platforms attain lower addictiveness and higher service quality when competing in prices. This result indicates that a platform may evaluate a business model in terms of how it affects the addictiveness of its service. For example, adopting a price-based revenue model may work as a commitment to less addictive features.

We close the paper with several directions for future research. First, the literature points to behavioral biases that are relevant to addiction, so it would be promising to incorporate them into a model of competition for attention in which firms may exploit behavioral biases to capture attention. Second, it appears worth studying platforms' choices of business models when they can choose addictiveness. From the empirical perspective, a natural question is what features of digital services correspond to the "addictiveness" of our model and how consumers' time allocation responds to various features of digital services. We anticipate further studies on various intriguing questions on theoretical and empirical fronts.

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Appendix

A Proof of Proposition 1

Proof. Suppose that the monopolist chooses $d^M(\overline{A}) := \min\left\{d^A(\overline{A}), d^P(\overline{A})\right\}$. Because $d^M(\overline{A}) \leq d^P(\overline{A})$, it is optimal for the consumer to join the platform. If $d^M(\overline{A}) = d^P(\overline{A})$ and the monopolist increases addictiveness, the consumer will not join it. If $d^M(\overline{A}) = d^A(\overline{A})$ and the monopolist increases addictiveness, the consumer will continue to choose $\overline{A}$ because her marginal utility of allocating attention is increasing in d. The monopolist then continues to earn the same payoff, $\overline{A}$. In either case the monopolist does not strictly benefit from changing $d^M(\overline{A})$. Because $d^P(\overline{A})$ and $d^A(\overline{A})$ are increasing in $\overline{A}$, $d^M(\overline{A})$ is increasing in $\overline{A}$.

To show the comparative statics in $\overline{A}$, we say that *the participation constraint binds* if the consumer's equilibrium payoff is zero. We also say that *the attention constraint is slack* if the consumer chooses $A(d^M(\overline{A})) \leq \overline{A}$, that is, the consumer's unconstrained choice of attention at $d^M(\overline{A})$ satisfies the attention constraint. Note that the attention constraint can hold with equality and be slack.

First, take any $\overline{A}^1$ at which the participation constraint binds. If the attention constraint is not slack (i.e., $A(d^M(\overline{A}^1)) > \overline{A}^1$), the platform could slightly lower addictiveness to attain the same payoff $\overline{A}^1$. This contradicts the tie-breaking rule of the monopolist (see Section 2). Thus if the participation constraint binds, the attention constraint is slack. As a result, we have $\max_{a\geq 0} u(a, d^M(\overline{A}^1)) - C(a) = 0$, i.e., the consumer's optimal payoff from the unconstrained problem is equal to zero. For any $\overline{A}^2 > \overline{A}^1$ we have

$$\max_{a\in[0,\overline{A}^2]} u(a, d^M(\overline{A}^2)) - C(a) \leq \max_{a\geq 0} u(a, d^M(\overline{A}^1)) - C(a) = 0,$$

because in the right-hand side of the inequality, the consumer does not face the attention constraint and the platform chooses lower addictiveness. As a result, the participation constraint also binds at $\overline{A}^2$. Thus there is some $\overline{A}^M$ such that the participation constraint binds if and only if $\overline{A} \geq \overline{A}^M$.

We show the comparative statics using $A(0)$ and $\overline{A}^M$. First, for any $\overline{A} \leq A(0)$ the consumer chooses $\overline{A}$ at $d^M = 0$, so it is indeed the monopolist's equilibrium choice. The consumer's equilibrium payoff is increasing in $\overline{A}$ whenever $\overline{A} \leq A(0)$ because the consumer faces the same addictiveness with a more relaxed attention constraint. Because the consumer earns a positive payoff at $d^M = 0$, we have $\overline{A}^M > A(0)$. Second, for any $\overline{A} \in [A(0), \overline{A}^M)$ the participation constraint is not binding. In such a case we have $d^M(\overline{A}) = d^A(\overline{A})$, i.e., the monopolist chooses the lowest addictiveness at which the consumer exhausts her attention. Given $d^A(\overline{A})$, the consumer's unconstrained choice $A(d^M(\overline{A}))$ satisfies the attention constraint with equality. To show that her payoff decreases on $[A(0), \overline{A}^M]$, take any $\overline{A}^3$ and $\overline{A}^4$ such that $A(0) \leq \overline{A}^3 < \overline{A}^4 < \overline{A}^M$. We have

$$\begin{aligned}
&\max_{a \in [0, \overline{A}^4]} \; u(a, d^M(\overline{A}^4)) - C(a) \\
=&\max_{a \geq 0} \; u(a, d^M(\overline{A}^4)) - C(a) \\
\leq&\max_{a \geq 0} \; u(a, d^M(\overline{A}^3)) - C(a) \\
=&\max_{a[0, \overline{A}^3]} \; u(a, d^M(\overline{A}^3)) - C(a)
\end{aligned}$$

Thus the consumer's payoff is lower under $\overline{A}^4$ than $\overline{A}^3$. Finally the consumer's payoff hits zero at $\overline{A} = \overline{A}^M$, which completes the proof. □

B Proof of Proposition 2

We first prove several lemmas.

Lemma B.1. *Take any increasing, strictly concave, and differentiable function, $u(\cdot) : \mathbb{R}_+ \to \mathbb{R}$. Then, $u(x) - xu'(x)$ is strictly increasing in x.*

Proof. For any x and $y > x$, we have

$$\frac{u(y) - u(x)}{y - x} > u'(y) \Rightarrow \; u(y) - u(x) > u'(y)(y - x) \Rightarrow \; u(y) - yu'(y) > u(x) - xu'(x).$$

□

Lemma B.2. *For any $y > 0$, consider the problem*

$$\max_{A \in [0, \overline{A}]} y \cdot u\left(\frac{A}{y}, d\right) - C(A). \tag{A.1}$$

Let $A^*(y)$ and $V^*(y)$ denote the maximizer and the maximized value, respectively. Then, $A^*(y)$ is increasing in y, $\frac{A^*(y)}{y}$ is decreasing in y, $V^*(y)$ is strictly concave in y, and $\frac{dV^*}{dy}$ is decreasing in d.

Proof. We write u_1 and u_{11} for $\frac{\partial u}{\partial a}$ and $\frac{\partial^2 u}{\partial a^2}$. Define $V(A, y) := y \cdot u\left(\frac{A}{y}, d\right) - C(A)$. We have $\frac{\partial^2 V}{\partial A \partial y} = -\frac{A}{y^2} u_{11}\left(\frac{A}{y}, d\right) > 0$, and thus $A^*(y)$ is increasing in y. To show $\frac{A^*(y)}{y}$ is decreasing, we rewrite (A.1) as

$$\max_{a \in [0, \overline{A}/y]} y \cdot u\left(a, d\right) - C(ay). \tag{A.2}$$

The maximizer of (A.2) is $a^*(y) := \frac{A^*(y)}{y}$. If $A^*(y) < \overline{A}$, then $a^*(y)$ satisfies the first-order condition $u_1(a,d) - C'(ay) = 0$, whose solution is decreasing in y. If y is so large that $A^*(y) = \overline{A}$, then for any such y, we have $a^*(y) = \frac{\overline{A}}{y}$, which is decreasing in y. Because $\frac{A^*(y)}{y}$ is continuous in y, it is decreasing in y.

We now show that $V^*(y)$ is concave. The envelope theorem (e.g., Corollary 4 of Milgrom and Segal (2002)) implies

$$\frac{dV^*}{dy} = u\left(\frac{A^*(y)}{y}, d\right) - \frac{A^*(y)}{y} u_1\left(\frac{A^*(y)}{y}, d\right).$$

This expression is decreasing in y, because $u(x,d) - xu'(x,d)$ is increasing in x (Lemma B.1) and $\frac{A^*(y)}{y}$ is decreasing in y. Finally, $\frac{\partial^2 V^*}{\partial y \partial d} = u_2\left(\frac{A^*(y)}{y}, d\right) + y \cdot u_{12}\left(\frac{A^*(y)}{y}, d\right) \cdot \frac{\partial}{\partial y}\left(\frac{A^*(y)}{y}\right) < 0$. The cross derivative $\frac{\partial^2 V^*}{\partial y \partial d}$ is well-defined for all $y \neq y^*$. Thus, $\frac{\partial V^*}{\partial y} = \int_0^d \frac{\partial^2 V^*}{\partial y \partial d}(y,t)dt + c$ (with some constant c) is decreasing in d. $\square$

The next lemma says that the incremental gain of joining a platform decreases in the addictiveness of other platforms the consumer has joined.

Lemma B.3. *Fix any $d' \geq 0$ and $\overline{A} > 0$, and consider the problem*

$$U(x,y,d) := \max_{(A_x, A_y) \in \mathbb{R}^2} x \cdot u\left(\frac{A_x}{x}, d\right) + y \cdot u\left(\frac{A_y}{y}, d'\right) - C(A_x + A_y) \tag{A.3}$$
$$s.t. \quad A_x \geq 0, A_y \geq 0, A_x + A_y \leq \overline{A}.$$

Then, $U_2(x,y,d)$ is decreasing in d.

Proof. The envelope theorem implies $U_2(x,y,d) = u\left(\frac{A_y^*}{y}, d'\right) - \frac{A_y^*}{y} u_1\left(\frac{A_y^*}{y}, d'\right)$, where A_y^* is a part of the maximizer of (A.3). The objective function in (A.3) is supermodular in $(A_x, -A_y, d)$, so A_y^* is decreasing in d. Lemma B.1 implies $u(a,d') - a \cdot u_1(a,d')$ is increasing in a. Thus $U_2(x,y,d)$ is decreasing in d. $\square$

The following result shows that the consumer faces a decreasing incremental gain of joining platforms for any choices of addictiveness.

Lemma B.4. *Take any $S, S' \subset K_{-1} := \{2, 3, \ldots, K\}$ such that $S' \subset S$. For any choice of addictiveness, the consumer's incremental gain of joining platform 1 is greater when she has already joined platforms S' than S. Formally, the following holds. Fix any $(d_1, \ldots, d_K) \in \mathbb{R}_+^K$. For any $y \in [0,1]$ and $S \subset K_{-1}$, define*

$$V(y,S) := \max_{(a_k)_{k \in S \cup \{1\}}} \sum_{k \in S} u(a_k, d_k) + y \cdot u(a_1, d_1) - C\left(\sum_{k \in S \cup \{1\}} a_k\right) \tag{A.4}$$
$$\textit{s.t.} \quad \sum_{k \in S \cup \{1\}} a_k \leq \overline{A} \quad \textit{and} \quad a_k \geq 0, \forall k \in S \cup \{1\}.$$

Then for any $S', S \subset K_{-1}$ such that $S' \subsetneq S$,

$$\frac{\partial V}{\partial y}(y,S) \leq \frac{\partial V}{\partial y}(y,S'). \tag{A.5}$$

In particular, $V(1,S) - V(0,S) \leq V(1,S') - V(0,S')$. These inequalities are strict whenever the consumer allocates positive attention to every platform in S and S' upon solving (A.4).

Proof. We fix any $(d_1, \ldots, d_K)$. To simplify notation, we write $u(a,d)$ as $u(a)$ and $\frac{\partial u}{\partial a}$ as $u'(a)$. Let $a_1(y,S)$ denote the optimal value of a_1 in (A.4). By the envelope formula, we have

$$\frac{\partial V}{\partial y}(y,S) = u(a_1(y,S)).$$

To show (A.5), we first show $a_1(y,S) \leq a_1(y,S')$ for any $S' \subset S$.

Suppose to the contrary that $a_1(y,S) > a_1(y,S')$, which implies $u'(a_1(y,S')) > u'(a_1(y,S))$. Because $a_1(y,S) > 0$, we have $u'(a_1(y,S)) \geq u'(a_j(y,S))$ for all $j \in S$; otherwise, the consumer can increase her payoff by decreasing a_1 and increasing a_j. Similarly, for every $j \in S'$ such that $a_j(y,S') > 0$, we have $u'(a_j(y,S')) \geq u'(a_1(y,S'))$. These inequalities imply that for every $j \in S'$ with $a_j(y,S') > 0$, we have $u'(a_j(y,S')) > u'(a_j(y,S))$, or equivalently, $a_j(y,S) > a_j(y,S')$. Also, there is some $j \in S'$ with $a_j(y,S') > 0$ because of the last inequality in Assumption 2. We derive a contradiction. First, if we have $\sum_{k \in S' \cup \{1\}} a_k(y,S') = \overline{A}$, we obtain $\sum_{k \in S \cup \{1\}} a_k(y,S) > \overline{A}$, which is a contradiction. Second, if $\sum_{k \in S' \cup \{1\}} a_k(y,S') < \overline{A}$, then for any j with $a_j(y,S') > 0$, we have

$$u'(a_j(y,S)) < u'(a_j(y,S')) = C'\left(\sum_{k\in S'\cup\{1\}} a_k(y,S')\right) < C'\left(\sum_{k\in S\cup\{1\}} a_k(y,S)\right),$$

which is also a contradiction. As a result, we obtain $a_1(y,S) \le a_1(y,S')$. Integrating both sides of (A.5) from $y=0$ to $y=1$, we have $V(1,S)-V(0,S) \le V(1,S')-V(0,S')$. If the consumer allocates positive attention to every platform in S and S' upon solving (A.4), we can use the same argument to show that $a_1(y,S) \ge a_1(y,S')$ (i.e., weak inequality) leads to a contradiction. Thus we have $a_1(y,S) < a_1(y,S')$ and obtain (A.5) as a strict inequality. □

Proof of Proposition 2. STEP 1: *There is a unique $d^* > 0$ that satisfies equation* (3). To show this, define

$$f(K,d) := Ku\left(\frac{A_K(d)}{K}, d\right) - C(A_K(d)) - \left[(K-1)u\left(\frac{A_{K-1}(d)}{K-1}, d\right) - C(A_{K-1}(d))\right].$$

The function $f(K,d)$ is the difference between payoffs when the consumer joins K platforms and when she joins $K-1$ platforms, given optimally allocating attention. Hereafter, we use the notation $V^*(y,d)$ for $V^*(y)$ of Lemma B.2 to make the dependence of $V^*(y)$ on d explicit. We can write $f(K,d) = V^*(K,d) - V^*(K-1,d)$. Lemma B.2 implies $V_1^*(y,d)$ is decreasing in d. Thus, $f(K,d) = \int_{K-1}^{K} V_1^*(y,d)dy$ is decreasing in d. Points (a) and (b) of Assumption 1 imply $f(K,0) > 0$ and $f(K,d) < 0$ for a large d. Thus, there is a unique positive d^* such that $f(K,d^*) = 0$, and d^* solves (3).

STEP 2: *There is an equilibrium in which each platform sets d^*.* Suppose all platforms choose d^*. First, we show that the consumer prefers to join all the platforms. Given $d_k = d^*$ for all k, the consumer's payoff from joining $J \le K$ platforms is $V^*(J,d^*)$, which is concave in J (Lemma B.2). We have $V^*(K,d^*) = V^*(K-1,d^*)$ by construction, which implies $V^*(k,d^*) - V^*(k-1,d^*) \ge 0$ for all $k \le K$. As a result, $V^*(k,d^*)$ is increasing in $k \le K-1$. Thus, the consumer prefers to join all platforms.

Second, no platform has a profitable deviation. Consider the incentive of (say) platform 1. If it increases d_1, the consumer joins only platforms $2,\dots,K$ to achieve the same payoff as without platform 1's deviation. Platform 1 does not benefit from such a deviation.

Suppose platform 1 decreases d_1 from d^* to d. The consumer joins platform 1. If she additionally joins other $y \leq K-1$ platforms, her payoff becomes $U(1, y, d)$ according to the notation of Lemma B.3 (with $d' = d^*$). Before the deviation (i,e., $d_1 = d^*$), $U(1, y, d^*)$ is maximized at $y = K-2$ and $y = K-1$. Because $U_{23}(1, y, d) < 0$ by Lemma B.3, the consumer's marginal gain from joining platforms increases after platform 1's deviation to $d < d^*$. As a result, $U(1, y, d)$ is uniquely maximized at $y = K-1$ across all $y \in \{1, \ldots, K-1\}$, so the consumer joins all platforms after the deviation. However, the consumer will then allocate a smaller amount of attention to platform 1 compared to without deviation, because platform 1 now offers a lower marginal utility. Thus, platform 1 does not strictly benefit from decreasing addictiveness.

STEP 3: *The uniqueness of equilibrium.* Take any pure-strategy subgame perfect equilibrium. Because any platform can set $d_k = 0$ to ensure participation, the consumer joins all platforms in equilibrium. First, we show all platforms choose the same addictiveness. Suppose to the contrary that there is an equilibrium in which platforms choose $(d_k^*)_{k \in K}$ such that (without loss) $d_2^* = \max_k d_k^* > \min_k d_k^* = d_1^*$. We show platform 1 has a profitable deviation. Suppose platform 1 deviates and increases its addictiveness to $d_1 = d_1^* + \varepsilon < d_2^*$. We show that the consumer joins platform 1 for a small ε. Suppose the consumer joins platform 2 after the deviation. Then she will also join platform 1; otherwise, she could obtain a strictly higher payoff by replacing platform 2 with 1. Suppose she does not join platform 2 after the deviation. Note that before the deviation, the consumer weakly prefers to join platform 1 when she joins other $K-1$ platforms. Lemma B.4 implies that before the deviation the consumer strictly prefers to join platform 1 when she does not join platform 2. As a result even after the deviation, the consumer strictly prefers to join platform 1 for a small $\varepsilon > 0$. In either case, platform 1 can profitably deviate to $d_1^* + \varepsilon$ with a small $\varepsilon > 0$ because the consumer joins platform 1 and allocates strictly greater attention. We obtain a contradiction because $(d_k^*)_{k \in K}$ is a part of equilibrium.

We have shown that all platforms choose the same addictiveness in any pure-strategy equilibrium. If the equilibrium addictiveness does not satisfy the consumer's indifference condition (3), then either (i) the left-hand side is strictly greater, in which case a platform prefers to deviate and increase its addictiveness, or (ii) the right-hand side is greater, in which case the consumer does not join at least one platform. In either case we obtain a contradiction. □

C Proofs for Section 4: The Impact of Competition

Proof of Proposition 4. In equilibrium, platforms choose positive addictiveness. In contrast, if the consumer exhausts her attention $\overline{A}$ at zero addictiveness, platforms choose $d = 0$ at the joint-profit maximizing outcome. Thus we obtain Points 1 and 2 if we show that the consumer chooses $\sum_{k \in K} a_k = \overline{A}$ under the conditions stated there.

First, let $A_K(0)$ denote the total attention the consumer would choose on K platforms with zero addictiveness. If $\overline{A} \leq A_K(0)$, she chooses $\sum_{k \in K} a_k = \overline{A}$, so we have Point 1. Second, the Inada-type condition in Point 2 implies that for some $\Delta > 0$, we have $u_1(\Delta, 0) - C'(\overline{A}) > 0$. For any $K > \frac{\overline{A}}{\Delta}$, the consumer will choose $\sum_{k \in K} a_k = \overline{A}$. Otherwise, we have $a_k < \Delta$ for at least one k, in which case the consumer could benefit from increasing her attention on platform k.

For Point 3, it suffices to show that the consumer obtains zero payoff at the joint-profit maximizing outcome for a large $\overline{A}$. Let $(d_1, \ldots, d_K)$ denote the solution of the joint-profit maximizing outcome when $\overline{A} = \infty$. Let A denote the total attention she will choose. Suppose she faces attention capacity $\overline{A} = B > A$ but obtains a positive payoff. Then the attention constraint must hold with equality. However, the platforms can attain $B > A$ at the joint-profit maximizing outcome, which contradicts the construction of A. Thus the consumer obtains zero payoff for any $\overline{A} > A$. □

D Proof of Proposition 5: Costly Investment in Addictive Technology

The appendix consists of several parts. First, we characterize the consumer's optimal attention allocation for any profile of addictiveness. Second, we characterize the equilibrium and the joint-profit maximizing outcome. Third, we provide a sufficient condition under which the consumer is better off under the joint-profit maximizing outcome. Finally, we prove Proposition 5. Without loss of generality, we assume $r = 1$, so platform k's payoff is $a_k - \kappa\gamma(d_k)$.

Lemma D.1. *Suppose that the consumer joins all of the K platforms with $(d_1, \ldots, d_K)$. Define*

$D = \sum_{k \in K} d_k$. *The consumer's optimal attention to platform k is*

$$a_k = \frac{1}{K} \left[\min\{A(D),\ \overline{A}\} - D \right] + d_k. \tag{A.6}$$

Here $A(\cdot) : \mathbb{R}_+ \to \mathbb{R}_+$ is concave and uniquely solves $A(D) - D = Kg\left(C'\left(A(D)\right)\right)$, where $g(\cdot)$ is the inverse of $v'(\cdot)$.

Proof. Having joined K platforms, the consumer chooses $(a_k)_{k \in K} \in \mathbb{R}_+^K$ to maximize

$$\sum_{k \in K} v(a_k - d_k) - C\left(\sum_{k \in K} a_k\right) \quad \text{s.t.} \quad \sum_{k \in K} a_k \leq \overline{A}.$$

The objective is concave, so the first-order condition characterizes the *unconstrained* optimal choice:

$$v'(a_k - d_k) - C'\left(\sum_{k \in K} a_k\right) = 0 \iff a_k - d_k = g\left(C'\left(\sum_{k \in K} a_k\right)\right). \tag{A.7}$$

Let $A = \sum_{k \in K} a_k$ and $D = \sum_{k \in K} d_k$. Summing up equation (A.7) across all $k \in K$, we obtain

$$A - D = Kg\left(C'\left(A\right)\right).$$

The left-hand side is strictly increasing in A and the right-hand side is strictly decreasing in A. Also, the left-hand is smaller if $A \leq D$ and is bigger for a large A. Thus there is a unique $A(D) > 0$ that solves the equation. Note that

$$A'(D) = \frac{1}{1 - Kg'\left(C'\left(A(D)\right)\right) \cdot C''\left(A(D)\right)} < 1. \tag{A.8}$$

The strict inequality in (A.8) holds because $g = (v')^{-1}$ is decreasing and $g'\left(C'\left(A(D)\right)\right) C''\left(A(D)\right)$ is negative. Under the assumption that $g'\left(C'\left(x\right)\right) \cdot C''\left(x\right)$ is decreasing in x, $A'(D)$ is decreasing in D.

We now show that the consumer's optimal total attention given the constraint is $\min\left\{A(D), \overline{A}\right\}$. Suppose $A(D) \leq \overline{A}$. We can directly verify that $a_k^* = \frac{1}{K}(A(D) - D) + d_k$ for each k satisfies the attention constraint and solves the consumer's first-order condition (A.7). Thus $(a_1^*, \ldots, a_K^*)$ is the optimal choice and satisfies $\sum_{k\in K} a_k^* = A(D)$. Suppose $A(D) > \overline{A}$. Suppose to the contrary that the consumer's optimal total attention A is strictly less than $\overline{A}$. Because $A < A(D)$, we have $A - D < Kg\left(C'\left(A\right)\right)$, which implies $v'(a_k - d_k) > C'(A)$ for some k. As a result, the consumer can slightly increase some a_k to increase her payoff, which is a contradiction. Thus $A = \overline{A}$. To sum up, the consumer's optimal total attention is $\min\left\{A(D), \overline{A}\right\}$.

The consumer's constrained choice solves $v'(a_k - d_k) - C'\left(\sum_{k\in K} a_k\right) - \lambda = 0$, where λ is the Lagrangian multiplier for the attention constraint. Because $a_k - d_k$ is constant across k at the optimum, we have $a_k - d_k = \frac{1}{K}(\sum_{k\in K} a_k - D)$. Thus $a_k = \frac{1}{K}\left[\min\left\{A(D), \overline{A}\right\} - D\right] + d_k$. $\square$

Lemma D.2. *There is a unique pure-strategy subgame perfect equilibrium in which all platforms choose the same positive addictiveness* $\min\{d^1(\kappa), d^2\}$. *Here,* $d^1(\kappa)$ *is a unique* d^1 *that satisfies*

$$d^1 \in \arg\max_{x\geq 0} \frac{1}{K}\left[\min\{A(x+(K-1)d^1),\ \overline{A}\} - x - (K-1)d^1\right] + x - \kappa\gamma(x), \tag{A.9}$$

and d^2 *is the equilibrium addictiveness at no cost benchmark, i.e.,* d^2 *makes the consumer indifferent between joining* K *and* $K-1$ *platforms. Also* $d^1(\kappa)$ *is decreasing,* $\lim_{\kappa\to 0} d^1(\kappa) = \infty$*, and* $\lim_{\kappa\to\infty} d^1(\kappa) = 0$*. Thus there is a* κ^E *such that the equilibrium addictiveness is* d^2 *if and only if* $\kappa \leq \kappa^E$.

Proof. We show that (A.9) has a unique solution d^1. For any $\kappa > 0$, define

$$\Pi(x, d) := \frac{1}{K}\left[\min\{A(x+(K-1)d),\ \overline{A}\} - x - (K-1)d\right] + x - \kappa\gamma(x), \tag{A.10}$$

which is a platform's profit when it chooses x, other platforms choose d, and the consumer joins all platforms and allocates attention optimally. Because $A(\cdot)$ is concave and $\gamma(\cdot)$ is strictly convex, $\Pi(x, d)$ is strictly concave in x and has decreasing differences in (x, d). Thus for each d, a platform has a unique best response $x(d)$ that is decreasing in d. If $\Pi_x(0, 0) \leq 0$, then we have $x(0) = 0$. If $\Pi_x(0, 0) > 0$, then we have $x(0) > 0$ and $x(d) < d$ for a sufficiently large d because $\lim_{d\to\infty} \gamma'(d) = \infty$. As a result, there is a unique positive d^1 that satisfies $x(d^1) = d^1$. Addictiveness d^1 solves (A.9).

In a unique equilibrium, platforms choose $\min\{d^1(\kappa), d^2\} > 0$. First suppose $d^1(\kappa) \leq d^2$. It is an equilibrium that all platforms choose $d^1(\kappa)$: If platform k unilaterally deviates and chooses

$d_k > d^2$, the consumer does not join k. If it chooses $d_k \in (d^1(\kappa),\ d^2)$, the consumer joins k; however, the platform earns a lower payoff because $\Pi_x(d_k,\ d^1(\kappa)) < 0$. If it chooses $d_k < d^1(\kappa)$, then because $d_k < d^2$, we have $\Pi_x(d_k,\ d^1(\kappa)) > 0$, so platform k does not benefit from such a deviation. Second suppose $d^1(\kappa) > d^2$. By a similar argument, we can show that no platform has a profitable deviation from d^2. The uniqueness follows the same argument. For example, if all platforms choose $d \in (d^1(\kappa), d^2)$, one platform can profitably deviate by slightly decreasing d. Finally, the assumptions on $\gamma(\cdot)$ and equation (A.9) imply that $d^1(\kappa)$ is decreasing, $\lim_{\kappa\to 0} d^1(\kappa) = \infty$, and $\lim_{\kappa\to\infty} d^1(\kappa) = 0$. These properties ensure the existence of κ^E. □

Lemma D.3. *There is a unique joint-profit maximizing outcome, in which platforms choose the same addictiveness* $\min\{d_J^1, d_J^2\}$. *Here,* d_J^1 *solves*

$$\max_{d\geq 0} \left[\min\{A(Kd),\ \overline{A}\} - K\kappa\gamma(d)\right], \tag{A.11}$$

and d_J^2 *is the unique level of addictiveness at which the consumer obtains a payoff of zero by joining all platforms.*

Proof. Because $\gamma(\cdot)$ is strictly convex, d_J^1 is unique. The rest of the proof follows the same logic as Proposition 1. Even though the platforms' gross revenue depends only on $\sum_{k\in K} d_k$, the joint-profit maximizing outcome implies that all platforms choose the same addictiveness, because they incur a strictly convex cost of increasing d. □

Lemma D.4. *If the equilibrium addictiveness is* $d^1(\kappa)$ *(see Lemma D.2), the consumer is better off under the joint-profit maximizing outcome than the equilibrium.*

Proof. Suppose the primitives are such that the equilibrium addictiveness is $d^1(\kappa)$. We consider two cases. First, suppose that the consumer's total attention is $\overline{A}$ in equilibrium. At the joint-profit maximizing outcome, platforms do not choose strictly higher addictiveness than the minimum level of addictiveness at which the consumer exhausts her attention $\overline{A}$. As a result, platforms choose lower addictiveness under the joint-profit maximizing outcome than the equilibrium.

Second, suppose that the consumer's total attention is strictly less than $\overline{A}$ in equilibrium, i.e., $A(Kd^1(\kappa)) < \overline{A}$. Then we can rewrite (A.9) as the first-order condition:

$$\frac{1}{K}(A'(Kd^1(\kappa)) - 1) + 1 - \kappa\gamma'(d^1(\kappa)) = 0. \tag{A.12}$$

Similarly, the joint-profit maximizing outcome (that ignores the consumer's participation incentive) is $d^J > 0$ that solves

$$A'(Kd^J) - 1 + 1 - \kappa\gamma'(d^J) = 0. \tag{A.13}$$

Equation (A.8) implies $A'(Kd) - 1 < 0$, so for any d we have

$$\frac{1}{K}(A'(Kd) - 1) + 1 - \kappa\gamma'(d) \geq A'(Kd) - 1 + 1 - \kappa\gamma'(d),$$

which implies $d^1(\kappa) \geq d^J$. The addictiveness under the joint-profit maximizing outcome is at most d^J, so the consumer is better off under the joint-profit maximizing outcome. □

We now prove Proposition 5.

Proof of Proposition 5. Let $U^J(\kappa)$ and $U^E(\kappa)$ denote the consumer's payoffs at the joint-profit maximizing outcome and the equilibrium. We consider two cases. First, suppose that $U^J(0) \geq U^E(0)$. Note that $U^J(\kappa)$ is increasing in κ. Take any $\kappa' > 0$. Suppose that at κ', the equilibrium addictiveness is $d^1(\kappa')$. Lemma D.4 implies that the consumer is better off under the joint-profit maximizing outcome. Suppose that at κ', the equilibrium addictiveness is d^2, i.e., it is determined by the consumer's participation constraint. Then for any $\kappa < \kappa'$, the equilibrium addictiveness continues to be d^2. Because $U^J(0) \geq U^E(0)$ at $\kappa = 0$, we have $U^J(\kappa') \geq U^E(\kappa')$ at $\kappa = \kappa'$. To sum up, if $U^J(0) \geq U^E(0)$, then $U^J(\kappa) \geq U^E(\kappa)$ for all $\kappa \geq 0$, so we have $\kappa^* = 0$.

Second, suppose $U^J(0) < U^E(0)$. The equilibrium addictiveness at $\kappa = 0$ is d^2 by Lemma D.4. Recall that κ^E is the cutoff such that $d^2 = d^1(\kappa^E)$. For any $\kappa \geq \kappa^E$, the equilibrium addictiveness is $d^1(\kappa)$, so Lemma D.4 implies $U^J(\kappa) \geq U^E(\kappa)$. For any $\kappa < \kappa^E$, $U^J(\kappa)$ is increasing in κ and $U^E(\kappa)$ is constant. Thus there is a $\kappa^* \leq \kappa^E$ such that $U^J(\kappa) \leq U^E(\kappa)$ if $\kappa \leq \kappa^*$ and $U^J(\kappa) \geq U^E(\kappa)$ if $\kappa \geq \kappa^*$. □

E Appendix for Remark 2

We consider the setting in Section 5, in which platforms incur increasing convex costs $\kappa\gamma(d)$ to set d. The consumer incurs attention cost $C(a) = \hat{C}(a) + \mathbf{1}_{\{a \geq \overline{A}\}} \cdot \frac{c}{2}(a - \overline{A})^2$, where $\hat{C}$ satisfies Assumption 2 and $\mathbf{1}_{\{a \geq \overline{A}\}}$ is the indicator function that takes value 1 or 0 if $a \geq \overline{A}$ or $a < \overline{A}$, respectively. The consumer does not face an additional attention constraint. If $c = \infty$, attention cost C subsumes the attention constraint $a \leq \overline{A}$.

Claim E.1. *Fix any* $\overline{A} > 0$ *that is small enough to satisfy* $u_1(\frac{\overline{A}}{K}, 0) - \hat{C}'(\overline{A}) \geq 0$. *For a sufficiently large* c*, consumer surplus is strictly greater under the (unique) joint-profit maximizing outcome than at the equilibrium.*

Proof. We fix all parameters except c. We write $c = \infty$ for the original setting in which the consumer faces attention cost $\hat{C}$ and attention constraint $A \leq \overline{A}$. If $c = \infty$ the consumer is strictly better off at the joint-profit maximizing outcome. The reason is as follows. First, at the joint-profit maximizing outcome, platforms can capture the maximum total attention $\overline{A}$ at $d = 0$. Because a higher d is costly, $d = 0$ uniquely maximizes the joint profits. Second, under competition, Proposition 5 implies that platforms set $d > 0$. Thus the consumer is strictly better off at the joint-profit maximizing outcome.

It suffices to show that the consumer surpluses under the joint-profit maximizing outcome and the equilibrium are continuous at $c = \infty$. Consider the joint-profit maximization. We can focus on strategies in which all platforms choose the same d. Having joined the (joint) platform, the consumer chooses total attention A to maximize $Ku(\frac{A}{K}, d) - C(A)$. The first-order condition is $u_1(\frac{A}{K}, d) - C'(A) = 0$. Because $u_1(\frac{\overline{A}}{K}, 0) - \hat{C}'(\overline{A}) \geq 0$, the consumer chooses $A \geq \overline{A}$. Thus we can write the consumer's attention allocation problem as choosing $x \geq 0$, where $A = \overline{A} + x$. In terms of x, her payoff is $Ku(\frac{\overline{A}+x}{K}, d) - C(\overline{A} + x) = Ku(\frac{\overline{A}+x}{K}, d) - \hat{C}(\overline{A} + x) - \frac{c}{2}x^2$. The first-order condition in x is $u_1(\frac{\overline{A}+x}{K}, d) - \hat{C}'(\overline{A} + x) - cx = 0$. Let $x(c, d)$ denote the solution x of this equation, which depends on (c, d). Differentiating the first-order condition with respect to d and arranging it in terms of $x(c, d)$, we obtain

$$x_2(c, d) = \frac{u_{12}\left(\frac{\overline{A}+x(c,d)}{K}, d\right)}{c + \hat{C}''\left(\overline{A} + x(c, d), d\right) - K^{-1}u_{11}\left(\frac{\overline{A}+x(c,d)}{K}, d\right)}.$$

Let $d^J(c)$ denote the addictiveness under the joint-profit maximizing outcome given c, and suppose that $d^J(c)$ does not converge to 0 when $c \to \infty$. That is, there is some $d' > 0$ such that $d^J(c_n) \geq d'$ for some sequence c_n that diverges to ∞ as $n \to \infty$. Because the consumer does not join platforms when d is too high, we can assume that platforms choose addictiveness of at most $\overline{d}$ for all c's. Along the sequence c_n, the platforms incur the cost of at least $K \cdot \kappa\gamma(d')$ to set addictiveness d'. However, the incremental gain of raising addictiveness is at most $x(c, \overline{d}) - x(c, 0)$, which goes to 0 as $c \to \infty$. Indeed, we have $x(c, \overline{d}) - x(c, 0) = \int_0^{\overline{d}} x_2(c, z)dz = \int_0^{\overline{d}} \frac{u_{12}\left(\frac{\overline{A}+x(c,y)}{K}, y\right)}{c+\hat{C}''(\overline{A}+x(c,y),y)-K^{-1}u_{11}\left(\frac{\overline{A}+x(c,y)}{K}, y\right)}dy$. Dominated convergence theorem implies

$$
\begin{aligned}
&\lim_{c\to\infty} \int_0^{\overline{d}} \frac{u_{12}\left(\frac{\overline{A}+x(c,y)}{K}, y\right)}{c+\hat{C}''(\overline{A}+x(c,y),y)-K^{-1}u_{11}\left(\frac{\overline{A}+x(c,y)}{K}, y\right)}dy \\
&= \int_0^{\overline{d}} \lim_{c\to\infty} \left[\frac{u_{12}\left(\frac{\overline{A}+x(c,y)}{K}, y\right)}{c+\hat{C}''(\overline{A}+x(c,y),y)-K^{-1}u_{11}\left(\frac{\overline{A}+x(c,y)}{K}, y\right)}\right] dy \\
&= 0.
\end{aligned}
$$

We thus obtain a contradiction. To sum up, as $c \to \infty$, the addictiveness at the joint-profit maximizing outcome converges to 0.

We can establish a similar continuity argument at $c = \infty$ for the case of competition. Let $d(c)$ denote the equilibrium addictiveness given c. We analogously define $d_1(c)$ and $d_2(c)$ via equations (3) and (A.9) in Lemma E.2, respectively. We have $d_1(c) \to d_1(\infty)$ and $d_2(c) \to d_2(\infty)$ as $c \to \infty$. Otherwise, either equation (3) or (A.14) fails because the consumer's optimal attention allocation is continuous at $c = \infty$. Thus we have $d(c) = \min(d_1(c), d_2(c)) \to \min(d_1^*, d_2^*) = d(\infty)$.

We have shown that the consumer surplus at the joint-profit maximizing outcome and competition are continuous at $c = \infty$. Therefore for a sufficiently large c, the consumer is strictly better off under the joint-profit maximizing outcome. □

F Appendix for Remark 3

In the main text, we have assumed that u is decreasing and $\frac{\partial u}{\partial a}$ is increasing in d. We now consider a model in which both terms are increasing in d. Formally, we assume that $u(a, d)$ is strictly increasing and strictly concave in a, and both $u(a, d)$ and $\frac{\partial u}{\partial a}(a, d)$ are increasing in d for every a. As to the sign of u, we only assume that for every $a > 0$, there is some d such that $u(a, d) > 0$. Thus, we allow $u(0, d) = 0$ for all d, or alternatively, $u(0, d) < 0$ for a small d. Hereafter we call d "quality."

There are two platforms, 1 and 2. If each platform chooses quality d and obtains attention a, the payoff is $a - \gamma(d)$, where $\gamma(d)$ satisfies the assumptions in Section 5. To exclude trivial outcomes, we assume that (u, γ) is such that the consumer joins both platforms and they earn positive payoffs in equilibrium and in the joint-profit maximizing outcome. As in Section 4, the consumer chooses between joining two platforms or joining none under the joint-profit maximizing game.

We define several objects. First, given (d_1, d_2), let $a_1(d_1, d_2)$ and $a_2(d_1, d_2)$ denote the amounts of attention the consumer allocates to platforms 1 and 2, respectively:

$$(a_1(d_1, d_2), a_2(d_1, d_2)) := \arg \max_{(a_1, a_2) \in \mathbb{R}^2_+} [u(a_1, d_1) + u(a_2, d_2) - C(a_1 + a_2)] \quad \text{subject to} \quad a_1 + a_2 \leq \overline{A}.$$

Let $U(d_1, d_2)$ denote the indirect utility of the consumer (i.e., the maximized value of the above problem) given (d_1, d_2). Let $\hat{U}(d)$ denote the indirect utility of the consumer when she joins a single platform that sets quality d.

We assume that the solution of the joint-profit maximizing problem is symmetric, i.e., $d_1 = d_2$, and the joint profits evaluated at the symmetric strategy profile of the platforms, i.e., $a_1(d, d) + a_2(d, d) - 2\gamma(d)$, is strictly concave in d. Having this in mind, let d_A^J solve

$$\max_{d \geq 0} a_1(d, d) + a_2(d, d),$$

and define $d_B^J = \min \{d \geq 0 : U(d, d) \geq 0\}$. Quality d_A^J maximizes the consumer's total attention conditional on that she joins the joint platform. Quality d_B^J is the minimum quality such that the consumer prefers to join the joint entity.

For each $k \in \{1, 2\}$, we write $-k$ for the unique element of $\{1, 2\} \setminus \{k\}$. As to the equilibrium, we assume that the equilibrium is unique and symmetric, and

$$\frac{\partial a_k}{\partial d_k}(d, d) - \gamma'(d) = 0 \tag{A.14}$$

has a unique positive solution, d_A^*. Also, let $d_B^* = \min \left\{ d \geq 0 : U(d, d) \geq \hat{U}(d) \right\}$. That is, d_B^* is the lowest quality such that the consumer weakly prefers joining both platforms to joining only one platform. We can apply Lemma B.2 to show that the consumer's incremental gain of joining a new platform is decreasing. Thus, at $(d_1, d_2) = (d_B^*, d_B^*)$, the consumer not only prefers joining two platforms to one platform, but also prefers joining one platform to joining none.

Claim F.1. *The joint-profit maximizing outcome is* $(d_1, d_2) = (\max(d_A^J, d_B^J), \max(d_A^J, d_B^J))$*, and the equilibrium outcome is* $(d_1, d_2) = (\max(d_A^*, d_B^*), \max(d_A^*, d_B^*))$*. The platform sets a strictly higher quality* d *and the consumer obtains strictly higher payoffs at equilibrium than the joint-profit-maximizing outcome.*

The intuition is as follows. When higher d increases both utilities and marginal utilities, competition encourages platforms to raise d to induce consumer participation and to capture more of the consumer's attention (i.e., business stealing incentive). Compared to the joint-profit maximizing outcome, both effects incentivize competing platforms to raise d. Thus regardless of C or $\overline{A}$, competition leads to higher quality and consumer surplus.

Proof. Suppose to the contrary that the joint-profit maximizing outcome (d^J, d^J), which we assume to be unique, is not $d^J = \max(d_A^J, d_B^J)$. Suppose $d_A^J \geq d_B^J$. If $d^J > d_A^J$, the joint entity can slightly decrease d^J to increase their profits. If $d^J \in [d_B^J, d_A^J)$, the joint entity can slightly increase d^J to increase their profits. If $d^J < d_B^J$, the consumer would not join the platforms, so the joint entity should set (say) d_B^J to ensure positive profits. We can apply a similar argument to the case of $d_A^J < d_B^J$ and obtains a contradiction. Thus $d^J = \max(d_A^J, d_B^J)$.

Next, suppose to the contrary that the equilibrium outcome (d^*, d^*), which we assume to be unique, is not $d^* = \max(d_A^*, d_B^*)$. If $d^* < d_B^*$, the consumer does not join at least one platform, which is a contradiction. Thus $d^* \geq d_B^*$. If $d^* > d_B^* \geq d_A^*$, then d^* does not satisfy (A.14), so one platform has a profitable deviation. Thus $d^* = d_B^*$. If $d_A^* > d_B^*$ and $d^* \neq d_A^*$, then d^* does not satisfy (A.14), so one platform can increase profits by slightly changing d_k while ensuring participation, which is a contradiction. Thus we have $d^* = \max(d_A^*, d_B^*)$.

We now show that platforms set a higher quality in equilibrium than in the joint-profit maximizing outcome. Suppose that the joint-profit maximizing outcome is d_B^J, which makes the consumer indifferent between joining two platforms and joining none. If $d_B^* \leq d_B^J$, the consumer weakly prefers joining no platform to joining both platforms at $(d_1, d_2) = (d_B^*, d_B^*)$. Because the consumer's indirect utility as a function of the number $K \in \{1, 2\}$ of platforms she joins is strictly concave (i.e., Lemma B.2), the consumer would strictly prefer joining a single platform to joining both platforms. This contradicts the definition of d_B^*, so we obtain $d_B^* > d_B^J$.

The first-order condition for d_k of the joint-profit maximizing outcome is

$$\frac{\partial a_k}{\partial d_k}(d_A^J, d_A^J) + \frac{\partial a_{-k}}{\partial d_k}(d_A^J, d_A^J) - \gamma'(d_A^J) = 0.$$

The first-order condition for d_k in equilibrium is

$$\frac{\partial a_k}{\partial d_k}(d_A^*, d_A^*) - \gamma'(d_A^*) = 0.$$

Suppose to the contrary that $d_A^J \geq d_A^*$. Adding up the first-order conditions for the joint-profit maximizing outcome, we have

$$\frac{\partial a_1}{\partial d_1}(d_A^J, d_A^J) + \frac{\partial a_2}{\partial d_2}(d_A^J, d_A^J) + \frac{\partial a_2}{\partial d_1}(d_A^J, d_A^J) + \frac{\partial a_1}{\partial d_2}(d_A^J, d_A^J) - \gamma'(d_A^J) - \gamma'(d_A^J) = 0.$$

Because the joint profit evaluated at $(d_1, d_2) = (d, d)$ is concave in d and we have $d_A^J \geq d_A^*$, we obtain

$$\frac{\partial a_1}{\partial d_1}(d_A^*, d_A^*) + \frac{\partial a_2}{\partial d_2}(d_A^*, d_A^*) + \frac{\partial a_2}{\partial d_1}(d_A^*, d_A^*) + \frac{\partial a_1}{\partial d_2}(d_A^*, d_A^*) - \gamma'(d_A^*) - \gamma'(d_A^*) \geq 0.$$

Because $a_k(d_k, d_{-k})$ is strictly decreasing in d_{-k}, we have $\frac{\partial a_2}{\partial d_1}(d_A^*, d_A^*) < 0$ and $\frac{\partial a_1}{\partial d_2}(d_A^*, d_A^*) < 0$. Thus we have

$$\frac{\partial a_1}{\partial d_1}(d_A^*, d_A^*) + \frac{\partial a_2}{\partial d_2}(d_A^*, d_A^*) - \gamma'(d_A^*) - \gamma'(d_A^*) > 0.$$

This inequality contradicts the first-order conditions for the equilibrium. Thus, $d_A^* > d_A^J$. Combining $d_A^* > d_A^J$ and $d_B^* > d_B^J$, we have $\max(d_A^*, d_B^*) > \max(d_A^J, d_B^J)$. Because $u(a, d)$ is increasing in d for every a, the consumer's payoff is strictly higher under competition than the joint-profit maximizing outcome. □

G Proof of Proposition 6: The Impact of Digital Curfew

Proof. Point 1 follows from Proposition 1, and Point 2 follows from Proposition 2. To show Point 3, for K, X, and d, let $A_K(X, d)$ denote the consumer's total attention when she faces attention cap $\overline{A} = X$ and joins K platforms with addictiveness d. Suppose the consumer initially faces the attention cap of $\overline{A} = A$. Let d^* denote the equilibrium addictiveness. Proposition 2 implies that the equilibrium addictiveness satisfies

$$K \cdot u\left(\frac{A_K(A, d^*)}{K}, d^*\right) - C\left(A_K(A, d^*)\right) = (K-1) \cdot u\left(\frac{A_{K-1}(A, d^*)}{K-1}, d^*\right) - C\left(A_{K-1}(A, d^*)\right).$$

Suppose that we decrease the attention cap to $X \in [A_{K-1}(A, d^*), A_K(A, d^*))$. We have $A_{K-1}(A, d^*) < A_K(A, d^*)$ because $A_K(A, d^*)$ is an interior solution by our assumption. Given $\overline{A} = X$ the attention constraint binds if she joins K platforms but not if she joins $K-1$ platforms. Thus we have

$$K \cdot u\left(\frac{X}{K}, d^*\right) - C\left(X\right) < (K-1) \cdot u\left(\frac{A_{K-1}(X, d^*)}{K-1}, d^*\right) - C\left(A_{K-1}(X, d^*)\right). \tag{A.15}$$

The incremental gain of joining a platform is decreasing in addictiveness (see the last part of Lemma B.2). Thus for any $d \geq d^*$, we have

$$K \cdot u\left(\frac{X}{K}, d\right) - C\left(X\right) < (K-1) \cdot u\left(\frac{A_{K-1}(X, d)}{K-1}, d\right) - C\left(A_{K-1}(X, d)\right). \tag{A.16}$$

Inequality (A.16) implies that if platforms increased addictiveness after a cap of X, the consumer joins at most $K-1$ platforms, which contradicts the equilibrium condition. Thus after a curfew the platforms set a strictly lower addictiveness.

Suppose a digital curfew decreases the attention cap from A to $A_D := A_{K-1}(A, d^*)$. If platforms continued to set d^*, this digital curfew would not change the consumer's payoff because she could join $K-1$ platforms and allocate attention $A_{K-1}(\overline{A}, d^*)$ optimally. After the cap, the platforms strictly decrease their addictiveness, so the consumer's payoff increases. $\square$

H Proofs for Section 7: Price Competition and Attention Competition

Proof of Lemma 1. We write $A_K(d)$ for the total attention the consumer chooses when she joins K platforms with addictiveness d. Define

$$p^* := Ku\left(\frac{A_K(0)}{K}, 0\right) - C(A_K(0)) - \left[(K-1)u\left(\frac{A_{K-1}(0)}{K-1}, 0\right) - C(A_{K-1}(0))\right].$$

We show that the game of price competition has an equilibrium in which each platform k sets $d_k = 0$ and $p_k = p^*$. Suppose each platform k sets $(d_k, p_k) = (0, p^*)$. The consumer chooses the number K' of platforms to join to maximize

$$V(K') := \max_{A \in [0, \overline{A}]} K' \hat{u}\left(\frac{A}{K'}, 0\right) - C(A) - K' p^*.$$

Lemma B.2 implies that $V(K')$ is concave on $[0, K]$. Because p^* makes the consumer indifferent between joining K and $K-1$ platforms, it is optimal for her to join all platforms. By the same argument as the case of attention competition, we can show that a platform does not strictly benefit from deviating to $p \neq p^*$.

The above equilibrium is unique. To show this, take any equilibrium and suppose each platform k chooses (d_k^*, p_k^*). First, we show that the consumer joins all platforms in equilibrium. Fix $\hat{k} \in K$, and suppose platform $\hat{k}$ sets $(d_{\hat{k}}, p_{\hat{k}}) = (0, 0)$, which may or may not be a deviation. Take any $K' \subset K$ such that $\hat{k} \notin K'$. First, if $d_j^* > 0$ for some $j \in K'$, then the consumer strictly prefers joining $(K' \setminus \{j\}) \cup \left\{\hat{k}\right\}$ to joining K'. Second, if $d_j^* = 0$ for all $j \in K'$ or $K' = \emptyset$, then the consumer strictly prefers $K' \cup \left\{\hat{k}\right\}$ to K'. Thus, for any set K' of platforms such that $\hat{k} \notin K'$, we can find some set S of platforms such that $\hat{k} \in S$ and the consumer strictly prefers S to K'. As a result, for a sufficiently small $p_{\hat{k}} > 0$ and $d_{\hat{k}} = 0$, the consumer still joins platform $\hat{k}$. Because any platform has a strategy to earn a positive profit, the consumer joins all platforms in any equilibrium.

Second, we show all platforms set zero addictiveness in any equilibrium. Suppose to the contrary that $d_k^* > 0$ for some k. Suppose platform k deviates and chooses $(d_k, p_k) = (0, p_k^*)$. Before the deviation, the consumer weakly prefers joining all platforms to joining any set K' of platforms that does not contain k. Thus, after the deviation to $(0, p_k^*)$, the consumer strictly prefers to joining platform k. As a result, platform k can slightly increase its price while retaining the consumer. This is a contradiction.

We have shown that in any equilibrium, the consumer joins all platforms, which set zero addictiveness. The price of each platform makes the consumer indifferent between joining and not joining the platform; otherwise, the platform can deviate by slightly increasing its price. Therefore, $(d_k^*, p_k^*) = (0, p^*)$ is a unique equilibrium. $\square$

Proof of Proposition 7. Take any $K \geq 2$ and let d^* denote the equilibrium addictiveness under attention competition. Note that because of $C(\cdot) \equiv 0$, the consumer allocates attention $\frac{\overline{A}}{K}$ to each platform under both attention competition and price competition. We then have

$$\begin{aligned}
&Ku\left(\frac{\overline{A}}{K}, 0\right) - K\left[Ku\left(\frac{\overline{A}}{K}, 0\right) - (K-1)u\left(\frac{\overline{A}}{K-1}, 0\right)\right] \qquad (A.17)\\
&= K(K-1)\left[u\left(\frac{\overline{A}}{K-1}, 0\right) - u\left(\frac{\overline{A}}{K}, 0\right)\right]\\
&< K(K-1)\left[u\left(\frac{\overline{A}}{K-1}, d^*\right) - u\left(\frac{\overline{A}}{K}, d^*\right)\right]\\
&= Ku\left(\frac{\overline{A}}{K}, d^*\right). \qquad (A.18)
\end{aligned}$$

The inequality holds because $u_{12} > 0$. The last equality follows from the equilibrium condition that the consumer is indifferent between joining K and $K-1$ platforms, i.e.,

$$Ku\left(\frac{\overline{A}}{K}, d^*\right) = (K-1)u\left(\frac{\overline{A}}{K-1}, d^*\right).$$

Expressions (A.17) and (A.18) are the consumer's equilibrium payoffs under price and attention competition, respectively. Thus the consumer is strictly better off under attention competition. □